BTETC402 Signals and Systems

Course Objectives:

- To understand the mathematical description of continuous and discrete time signals and systems.
- To classify signals into different categories.
- To analyze Linear Time Invariant (LTI) systems in time and transform domains.
- To build basics for understanding of courses such as signal processing, control system and communication.

Course Outcomes:

On completion of the course, students will be able to:

- Understand mathematical description and representation of continuous and discrete time signals and systems.
- Develop input output relationship for linear shift invariant system and understand the convolution operator for continuous and discrete time system.
- Understand and resolve the signals in frequency domain using Fourier series and Fourier transforms.
- Understand the limitations of Fourier transform and need for Laplace transform and develop the ability to analyze the system in s-domain.

UNIT – 1 Introduction to Signals and Systems:

Introduction and Classification of signals: Definition of signal and systems, Continuous time and discrete time signal,

Classification of signals as even, odd, periodic and non-periodic, deterministic and nondeterministic, energy and power

elementary signals used for testing: exponential, sine, impulse, step and its properties, ramp, rectangular, triangular, signum, sinc

Operations on signals: Amplitude scaling, addition, multiplication, differentiation, integration (Accumulator for DT), time scaling, time shifting and time folding,

Sampling Theorem and reconstruction of sampled signal, Concept of aliasing, examples on under sampled and over sampled signals.

Systems: Definition, Classification: linear and non-linear, time variant and invariant, causal and non-causal, static and dynamic, stable and unstable, invertible.

UNIT – 2 Time domain representation of LTI System:

System modelling: Input-output relation, definition of impulse response, convolution sum, convolution integral, computation of convolution integral using graphical method, Computation of convolution sum. Properties of convolution, properties of the system based on impulse response, step response in terms of impulse response.

UNIT – 3 Fourier Series:

Fourier series (FS) representation of periodic Continuous Time (CT) signals, Dirichlet condition for existence of Fourier series, FS representation of CT signals using exponential Fourier series, Fourier spectrum representation, properties of Fourier series, Gibbs phenomenon, Discrete Time Fourier Series and its properties.

UNIT – 4 Fourier Transform:

Fourier Transform (FT) representation of aperiodic CT signals, Dirichlet condition for existence of Fourier transform, evaluation of magnitude and phase response, FT of standard CT signals, FT of standard periodic CT signals, Introduction to Fourier Transform of DT signals, Properties of CTFT and DTFT, Fourier Transform of periodic signals. Concept of sampling and reconstruction in frequency domain, sampling of bandpass signals.

UNIT – 5 Laplace and Z-Transform:

Definition of Laplace Transform (LT), Limitations of Fourier transform and need of Laplace transform, ROC and its properties, properties of Laplace transform, Laplace transform evaluation using properties, Inverse Laplace transform based on partial fraction expansion, Application of Laplace transforms to the LTI system analysis.

Introduction to Z-transform, and its properties, Inverse Z-transform, different methods of inverse Z-transform, Z-transform for discrete time system LTI analysis.

TEXT/REFERENCE BOOKS:

1.Alan V. Oppenheim. Alan S. Willsky and S. Hamid Nawab, “Signals and Systems”, PHI

2.Dr. S. L. Nalbalwar, A.M. Kulkarni and S.P. Sheth, “Signals and Systems”, 2nd Edition, Synergy Knowledgeware,2017

3.Simon Haykins and Barry Van Veen, “Signals and Systems”, 2nd Edition, WileyIndia.

4.Shaila Apte, “Signals and Systems-principles and applications”, Cambridge University press,2016.

5.Mrinal Mandal and Amir Asif, Continuous and Discrete Time Signals and Systems, Cambridge University Press,2007.
6.Peyton Peebles, "Probability, Random Variable, Random Processes", 4th Edition, Tata McGraw Hill.
7.A. NagoorKanni "Signals and Systems", 2nd edition, McGrawHill
8.NPTEL video lectures on Signals andSystems.
9.Roberts, M.J., "Fundamentals of Signals & Systems", Tata McGraw Hill.2007.
10.Ziemer, R.E., Tranter, W.H. and Fannin, D.R., "Signals and Systems: Continuous and Discrete", 4th 2001 Ed., Pearson Education.

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1. Define signal.
2. Define system.
3. What are the major classifications of the signal.
4. Define discrete time signals and classify them.
5. Define continuous time signals and classify them.
6. Define discrete time unit step and unit impulse.
7. Define continuous time unit step and unit impulse.
8. Define unit ramp signal.
9. Define periodic signal and non-periodic signal.
10. Define even and odd signal.
11. Define energy and power signal.
12. Define unit pulse function.
13. Define continuous time complex exponential signal.
14. What is continuous time real exponential signal.
15. What is continuous time growing exponential signal?
16. What is continuous time decaying exponential?
17. What are the types of Fourier series? Write down the exponential form of the Fourier series representation of a periodic signal?
18. Write down the trigonometric form of the Fourier series representation of a Periodic signal?
19. Write short notes on dirichlets conditions for Fourier series.
20. State Time Shifting property in relation to Fourier series.
21. State Parseval's theorem for continuous time periodic signals.
22. Explain in detail discrete time signal and continuous time signal.
23. Explain in detail complex exponential CT signal.
24. Find the energy of the signal $x[n] = (1/2)^n u[n]$
25. Find the odd and even components of the signal: $cost + sin\ t + cost\ sin\ t$.
26. Find odd and even components of $x[n] = \{1, 2, 2, 3, 4\}$.
27. Find the energy of the signal $e^{-2t} u(t)$.
28. Test whether the signal $y(t) = ax(t) + b$ is linear or nonlinear.
29. Find power and rms value of the signal: $x(t) = 20cos2\pi t$
30. Explain the following signals
    i. Periodic and aperiodic
    ii. Even and odd.

FOURIER SERIES & Fourier Transform

31. Find the Fourier series coefficients for the following signal.

$$x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right)$$

32. Find the Fourier series coefficients for the following signal.

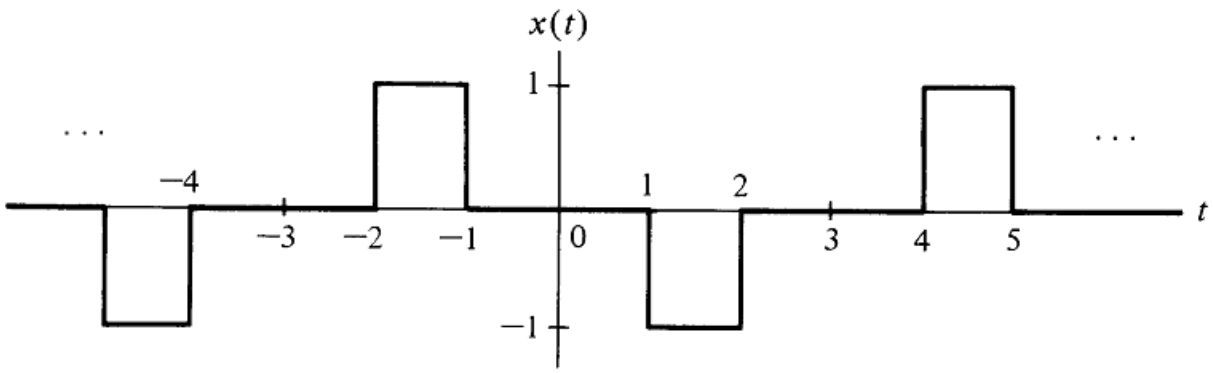
$$x(t) = 1 + \cos(2\pi t)$$

33. Find the Fourier series coefficients for the following signal.

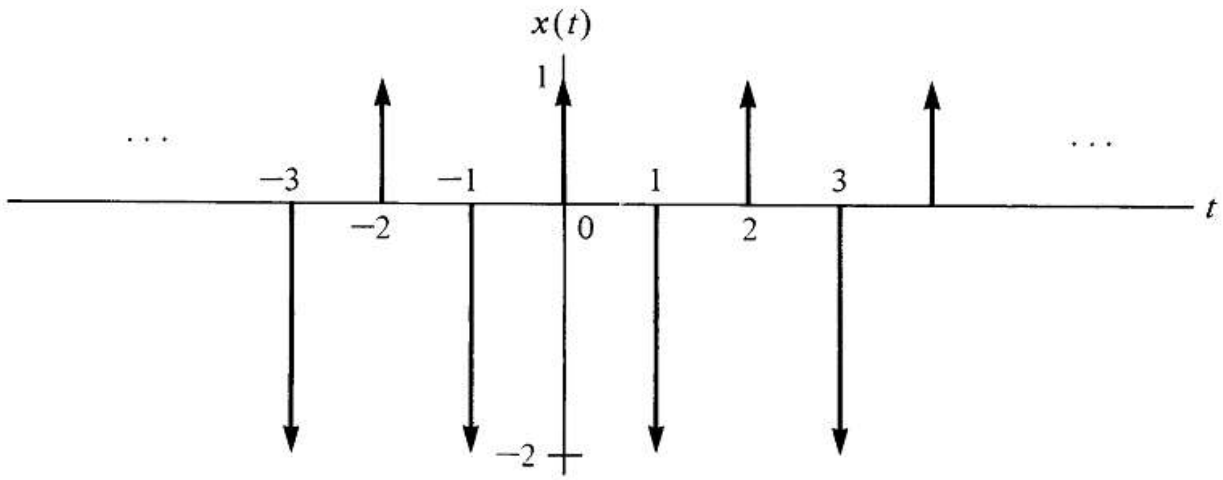
$$x(t) = [1 + \cos(2\pi t)]\left[\sin\left(10\pi t + \frac{\pi}{6}\right)\right]$$

Hint: You may want to first multiply the terms and then use appropriate trigonometric identities.

34. By evaluating the Fourier series analysis equation, determine the Fourier series for the following signal.



35. By evaluating the Fourier series analysis equation, determine the Fourier series for the following signal.



36. Without explicitly evaluating the Fourier series coefficients, determine which of the periodic waveforms in Figures (E) to (G) have Fourier series coefficients with the following properties.
   i. Has only odd harmonics
   ii. Has an only purely real coefficient.
   iii. Has an only purely imaginary coefficient.

a)

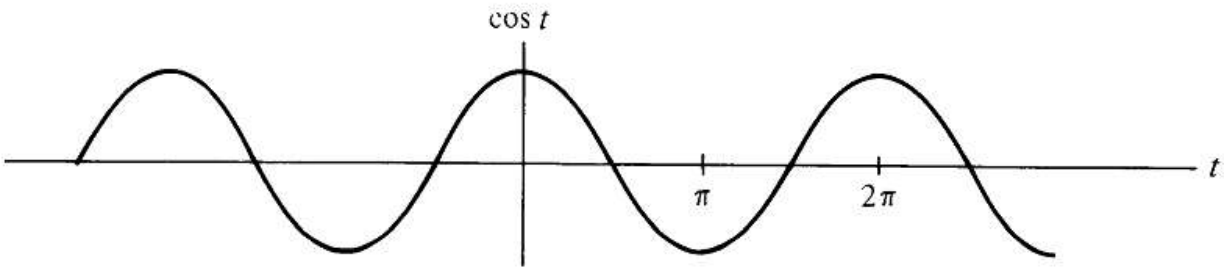


Figure (E)

b)

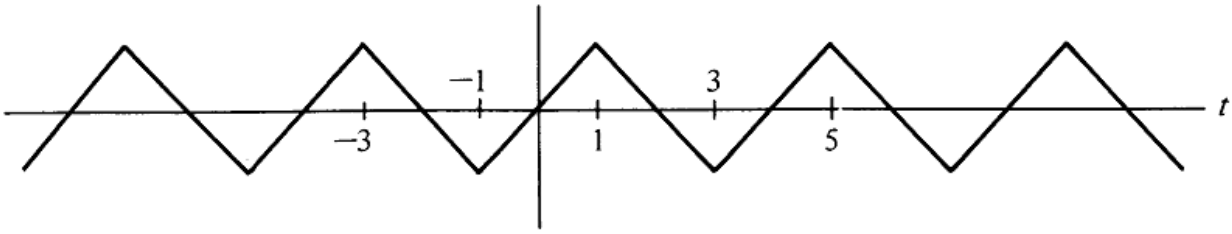


Figure (F)

c)

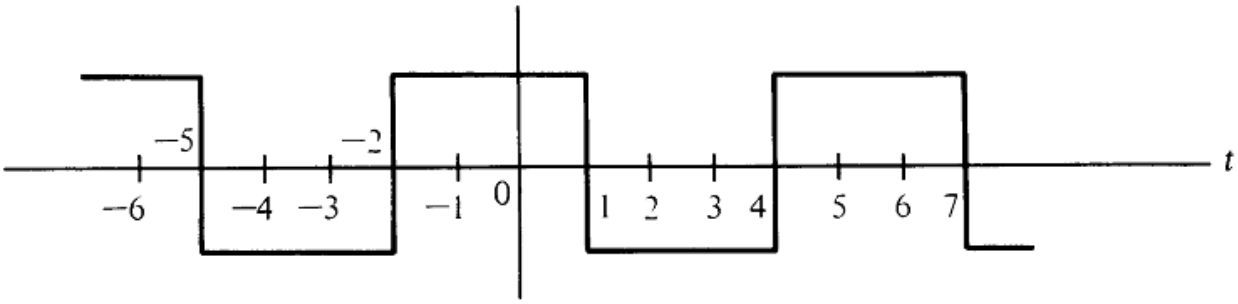


Figure (G)

37. Suppose x(t) is periodic with period *T* and is specified in the interval $0 < t < T/4$ as shown in Figure (H)

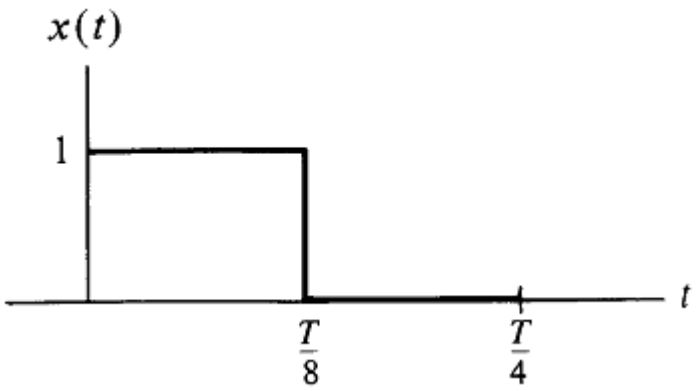


Figure (H)

Sketch *x(t)* in the interval **0** < *t* < *T* if

i. The Fourier series has only odd harmonics and x(t) is an even function.

ii. The Fourier series has only odd harmonics and x(t) is an odd function.

38. Find the Fourier transform of each of the following signal and sketch the magnitude and phase as a function of frequency, including both positive and negative frequencies.

$$\delta(t-5)$$

39. Find the Fourier transform of each of the following signal and sketch the magnitude and phase as a function of frequency, including both positive and negative frequencies.

$e^{-\alpha t}u(t)$ a real, positive

40. Find the Fourier transform of each of the following signal and sketch the magnitude and phase as a function of frequency, including both positive and negative frequencies.

$$e^{(-1+j2)t}\ u(t)$$

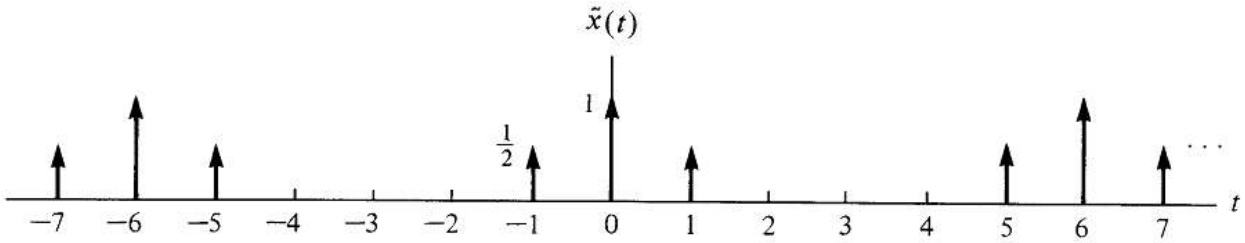
41. Show that if $x_3(t) = ax_1(t) + bx_2(t)$, then $X_3(w) = aX_1(w) + bX_2(w)$.

42. Verify that

$$e^{j\omega_0 t} = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\delta(\omega-\omega_0)e^{j\omega t}\,d\omega$$

From this observation, argue that the Fourier transform of $e^{jw_0 t}$ is $2\pi\delta(w - wo)$.

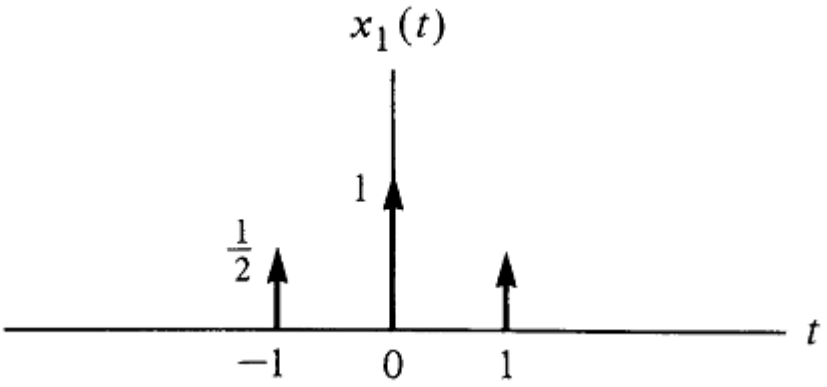
43. Consider the periodic signal t(t) in Figure **,** which is composed solely of impulses



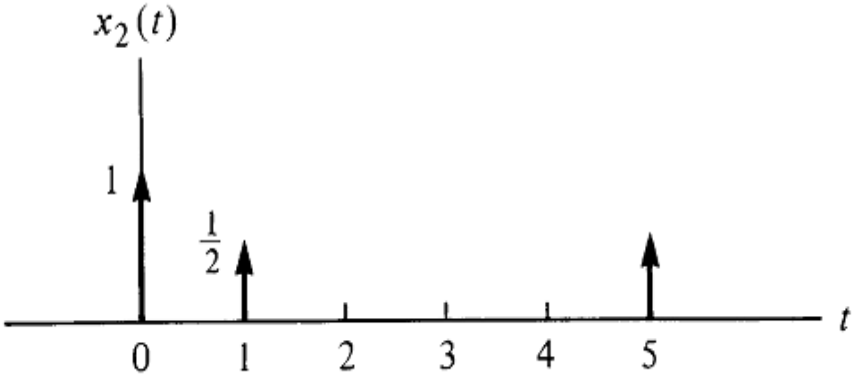
i. What is the fundamental period T_o?

ii. Find the Fourier series of $\tilde{x}(t)$.

44. Find the Fourier transform of the signals



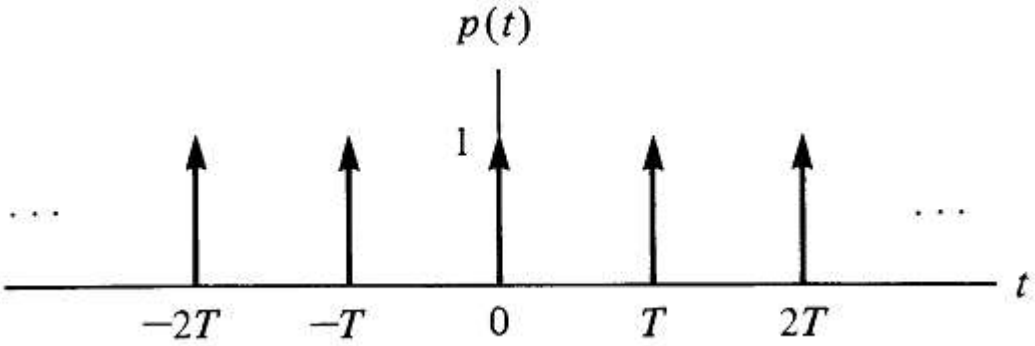
45. Find the Fourier transform of the signals shown in below figure.



46. Consider the impulse train

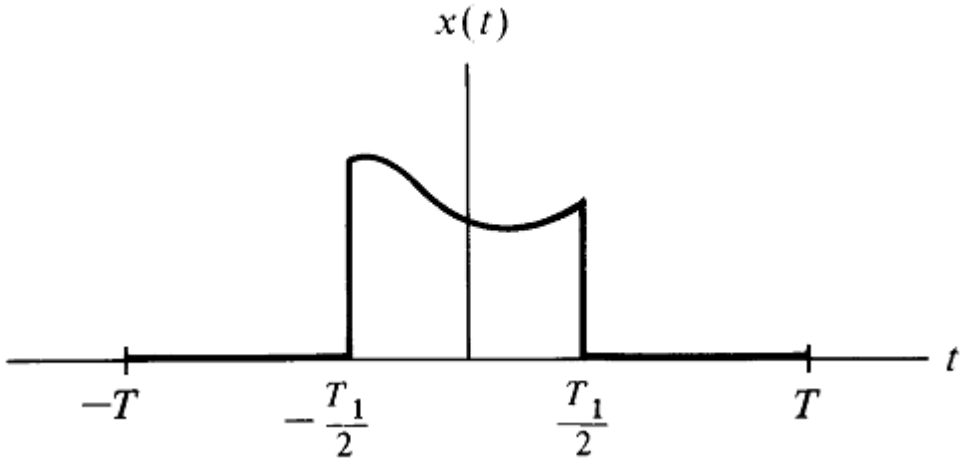
$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

as shown in below figure.



i. Find the Fourier series of p(t).
ii. Find the Fourier transform of p(t).

47. Consider the signal x(t) shown in below Figure, where $T_1 < T$.



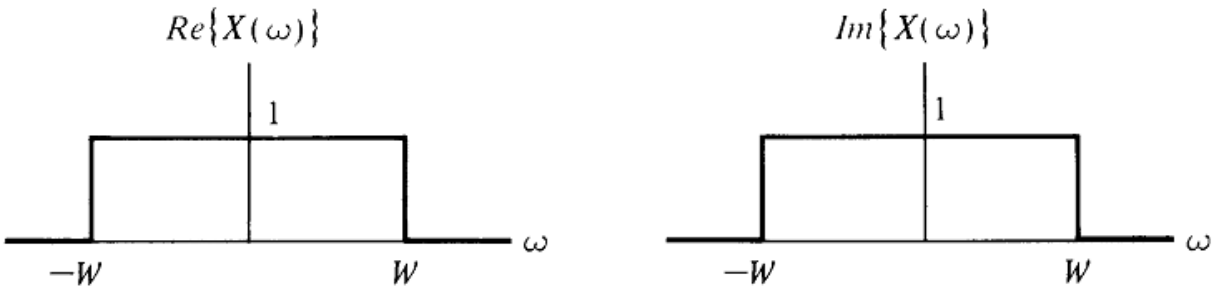
Show that the periodic signal t(t), formed **by** periodically repeating x(t), satisfies

$$\tilde{x}(t) = x(t) * p(t)$$

48. Determine the Fourier transform of $x(t) = e^{-\frac{t}{2}} u(t)$ and sketch

i. $|X(w)|$
ii. *Re { X (w) }*
iii. *Im { X (w) }*

49. Following figure shows real and imaginary parts of the Fourier transform of a signal *x(t).*



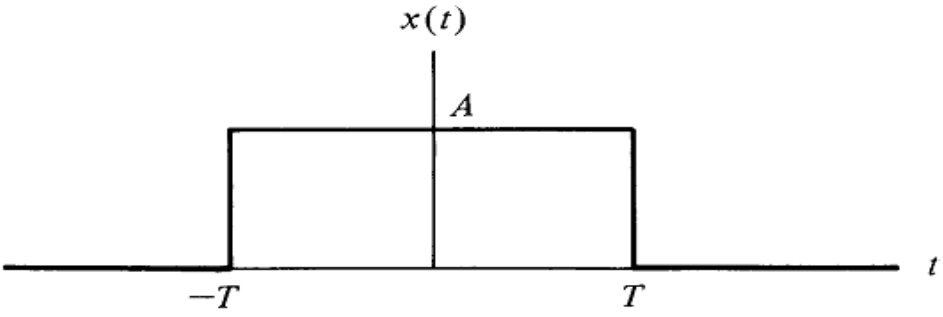
Sketch the magnitude and phase of the Fourier transform X (W).

50. If

$$r(t) = \frac{1}{1 + (3t)^2},$$

Find R (w)

51. *x (t)* is sketched in below figure **.** If y (t) = x (t/2), sketch y (t), *Y (w), and X (w).*



52. Compute the Fourier transform of each of the following signals

$$[e^{-\alpha t}\cos w_0 t]\, u(t), \qquad \alpha > 0$$

53. Compute the Fourier transform of each of the following signals

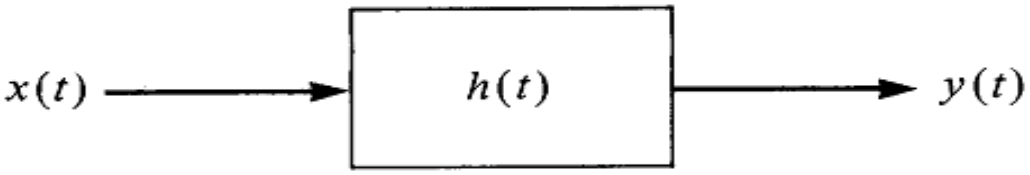
$$e^{-3|t|}\sin 2t$$

54. Compute the Fourier transform of each of the following signals

$$\left(\frac{\sin \pi t}{\pi t}\right)\left(\frac{\sin 2\pi t}{\pi t}\right)$$

55. From below figure find y(t) where

$$x(t) = \frac{\sin\ (w_0 t)}{t} \qquad and \qquad h(t) = \frac{\sin\ (2w_0 t)}{t}$$



56. Consider a discrete-time system with impulse response

$$h[n] = (1/2)^n U[n]$$

Determine the response to the following input.

$x[n] = (-\mathbf{1})^{\boldsymbol{n}}\, e^{j\pi n}$ For all n

57. Consider a discrete-time system with impulse response

$$h[n] = (1/2)^n U[n]$$

Determine the response to the following input.

$x(n) = e^{(\frac{j\pi n}{4})}$ For all n

58. Consider a discrete-time system with impulse response

$$h[n] = (1/2)^n U[n]$$

Determine the response to the following input.

$x[n] = \cos\ (\frac{\pi n}{4} + \frac{\pi}{8})$ For all n

59. Consider the following two periodic sequences:

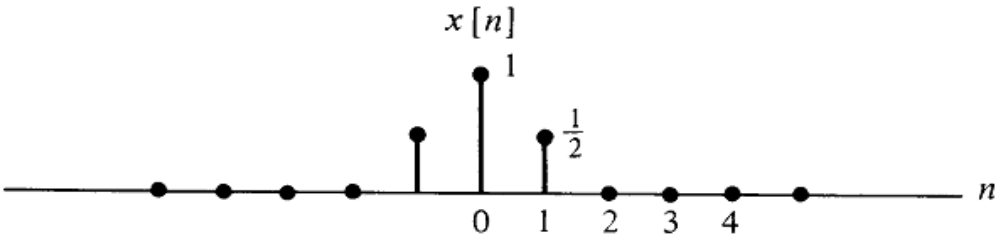
$$\tilde{x}_1[n] = 1 + \sin\left(\frac{2\pi n}{10}\right)$$

$$\tilde{x}_2[n] = 1 + \sin\left(\frac{20\pi}{12}n + \frac{\pi}{2}\right)$$

i. Determine the period of $x_1[n]$ and of $x_2[n]$

ii. Determine the sequence of Fourier series coefficients a_1k for $x_1[n]$ and a_2k for $x_2[n]$.

60. Determine and sketch the discrete-time Fourier transform of the sequence of figure as given below.



61. Consider a linear, time-invariant system with impulse response

$$h(n) = \left(\frac{1}{2}\right)^{|n|}$$

Find the Fourier series representation of the output $\tilde{y}(n)$ for the following input.

$$\tilde{x}(n) = \sin(\frac{3\pi n}{4})$$

62. Consider a linear, time-invariant system with impulse response

$$h(n) = \left(\frac{1}{2}\right)^{|n|}$$

Find the Fourier series representation of the output $\tilde{y}(n)$ for the following input.

$$\tilde{x}(n) = \sum_{k=-\infty}^{\infty} \delta(n-4k)$$

63. Consider a linear, time-invariant system with impulse response

$$h(n) = \left(\frac{1}{2}\right)^{|n|}$$

Find the Fourier series representation of the output $\tilde{y}(n)$ for the following input.

$$\tilde{x}(n) = (j)^n + (-1)^n$$

64. Determine the Fourier series coefficients for each of the following periodic discrete-time signals. Plot the magnitude and phase of each set of coefficients a_k.

$$x(n) = \sin[\frac{\pi(n-1)}{4}]$$

65. Determine the Fourier series coefficients for each of the following periodic discrete-time signals. Plot the magnitude and phase of each set of coefficients a_k.

$$x(n) = \cos\left(\frac{2\pi n}{3}\right) + \sin\left(\frac{2\pi n}{7}\right)$$

66. Determine the Fourier series coefficients for each of the following periodic discrete-time signals. Plot the magnitude and phase of each set of coefficients a_k.

$$x(n) = \cos\left(\frac{11\pi n}{4} - \frac{\pi}{3}\right)$$

67. Compute the discrete-time Fourier transform of the following signal.

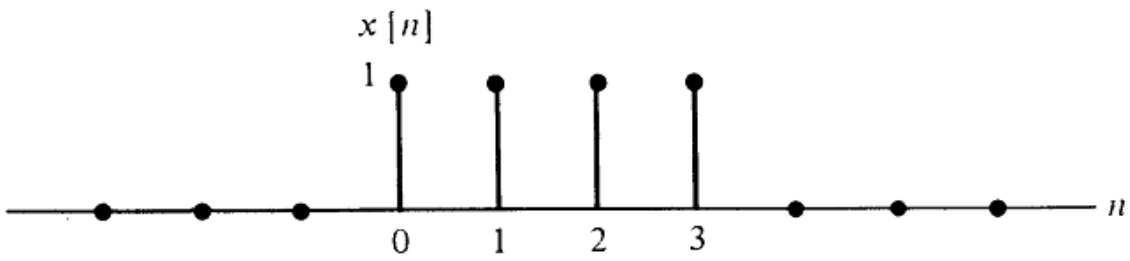
x [n] = $(1/4)^n$ u[n]

68. Compute the discrete-time Fourier transform of the following signal.

x [n] = (a^n sin $w_0 n$) u[n], $|a| < 1$

69. Compute the discrete-time Fourier transform of the following signal.

x [n] as shown in below figure.



70. Compute the discrete-time Fourier transform of the following signal

x [n] = $(1/4)^n$ u[n+2]

71. Using Fourier transforms, evaluate y[n] if x[n]=δ[n]

72. Using Fourier transforms, evaluate y[n] if x[n]=δ[n-n_0]

73. Using Fourier transforms, evaluate y[n] if x[n]=$(3/4)^n$ u[n]

74. Consider a system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^n \cos\ (\pi n/2)\, u[n]$$

i. Determine the system transfer function H (Ω).

ii. Suppose that $x[n] = cos(\pi n/2)$. Determine the system output $y[n]$ using the transfer function H (Ω) found in part (i).

75. A particular LTI system is described by the difference equation

$$y[n] + \tfrac{1}{4}y[n-1] - \tfrac{1}{8}y[n-2] = x[n] - x[n-1]$$

i. Find the impulse response of the system

ii. Evaluate the magnitude and phase of the system frequency response at $\Omega = 0$, $\Omega = \pi/4$, $\Omega = -\pi/4$, and $\Omega = 9\pi/4$.

76. Suppose we have an LTI system characterized by an impulse response

$x(n) = \sin(\pi n/3)/\pi n$

i. Sketch the magnitude of the system transfer function.

ii. Evaluate y[n] = x[n] $*$ h[n] when $x[n] = cos(3\pi n/4)$

77. A particular discrete-time system has input x [n] and output y[n]. The Fourier transforms of these signals are related by the following equation:

$$Y(\Omega) = 2X(\Omega) + e^{-j\Omega}X(\Omega) - \frac{dX(\Omega)}{d\Omega}$$

i. Is the system linear? Clearly justify your answer.

ii. Is the system time-invariant? Clearly justify your answer.

iii. What is y[n] if x[n] = δ [n]?

Laplace Transform

78. Determine the Laplace transform, pole and zero locations, and associated ROC for following time function.

i. $e^{-\alpha t}u(t), \quad a > 0$

ii. $e^{-\alpha t}u(t), \quad a < 0$

79. Determine the Laplace transform, pole and zero locations, and associated ROC for following time function.

i. x (t) = $3e^{2t}u(t) + 4e^{3t}u(t)$.

ii. $-e^{-\alpha t}u(-t), \quad a < 0$

80. Determine x(t) for the following conditions if X(s) is given by

$X(s) = \frac{1}{(s+1)(s+2)}$

i. x(t) is right-sided

ii. x(t) is left-sided

81. An LTI system has an impulse response h(t) for which the Laplace transform $H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}\,dt = \frac{1}{s+1}$ Re{s}> -1

Determine the system output y(t) for all t if the input x(t) is given by

$$x(t) = e^{-t/2} + 2e^{-t/3} \quad \text{for all t.}$$

82. Determine the Laplace transform, pole and zero locations, and associated ROC for following time function.

i. $u(t)$

ii. $\delta(t - t_0)$

iii. $cos\ (w_0 t + b)u(t)$

iv. $sin\ (w_0 t + b)e^{-\alpha t}u(t) \qquad a > 0$

83. If x(t) is an even time function such that x(t) = x(- t), show that this requires

that X(s) = X(-s).

84. If x(t) is an odd time function such that x(t) = -x(--t), show that

X(s) = -X (-s).

Z-Transform

85. An LTI system has an impulse response h[n] for which the z-transform is

$$H(Z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n} = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

Plot the pole-zero pattern for H(z).

86. Determine the z-transforms of the following two signals.

i. $x_1[n] = (1/2)^n u[n]$

ii. $x_2[n] = -\left(\frac{1}{2}\right)^n u[-n-1]$

iii. Sketch the pole-zero plot and ROC for each signal in part (i) and (ii)

87. Determine the z-transforms of the following two signals.

i. $x_3[n] = 2u[n]$

ii. $x_4[n] = -(2)^n u[-n-1]$

iii. Sketch the pole-zero plot and ROC for each signal in part (i) and (ii)

88. Determine the z-transform (including the ROC) of the following sequences. Also Sketch the pole-zero plots and indicate the ROC on your sketch.

i. $\left(\frac{1}{3}\right)^n u(n)$

ii. $\delta(n+1)$

89. Determine the inverse z-transform for following z-transform

$$X(z) = \frac{1}{1+\frac{1}{2}z^{-1}} \qquad |z| > \frac{1}{2}$$

90. Determine the inverse z-transform for following z-transform

$$X(z) = \frac{1-\frac{1}{2}z^{-1}}{1-\frac{1}{4}z^{-2}} \qquad |z| > \frac{1}{2}$$

91. Determine the inverse z-transform for following z-transform

$$X(z) = \frac{1-\alpha z}{z^{-1}-\alpha} \qquad |z| > \frac{1}{\alpha}$$

92. Suppose X(z) on the circle $z = 2e^{j\Omega}$ is given by

$$X(2e^{j\Omega}) = \frac{1}{1-\frac{1}{3}e^{-j\Omega}}$$

Using the relation $X(re^{j\Omega}) = F\{r^{-n}x(n)\}$, find $2^{-n}x(n)$ and then $x(n)$, the inverse z-transform of X (z).

93. Find x[n] from X(z) below using partial fraction expansion, where x[n] is known to be causal

$$X(z) = \frac{3+2z^{-1}}{2+3z^{-1}+z^{-2}}$$

94. Determine the z-transform for the following sequences. Express all sums in closed

form. Sketch the pole-zero plot and indicate the ROC. Indicate whether the Fourier

Transform of the sequence exists.

i. $(\frac{1}{2})^n\{u[n]-u[n-10]\}$

ii. $(\frac{1}{2})^{|n|}$

iii. $7(\frac{1}{3})^n[\cos\frac{2\pi n}{6}+\frac{\pi}{4}]$

95. Using the power-series expansion

$\log(1-w) = -\sum_{i=1}^{\infty}\frac{w^i}{i}$, $|w|$ <1

Determine the inverse of the following z-transforms.

i. $X(z) = \log(1-2z)$, $|z| < 1/2$

ii. $X(z) = \log(1-\frac{1}{2}z^{-1})$, $|z| > 1/2$

Introduction to Signal and System

96. Sketch each of the following signals.

i. $x[n] = \delta[n] + \delta[n-3]$

ii. $x[n] = u[n] - u[n-5]$

iii. $x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + (\frac{1}{2})^2\delta[n-2] + (\frac{1}{2})^3\delta[n-3]$

iv. $x(t) = u(t+3) - u(t-3)$

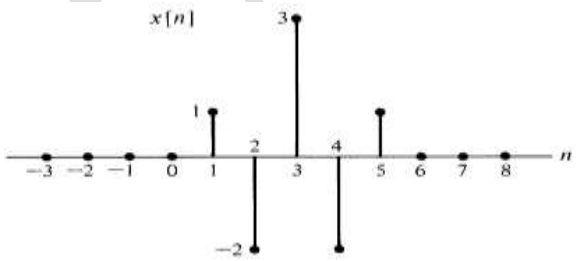
v. $x(t) = e^{-t}u(t)$

97. Below are two columns of signals expressed analytically? For each signal in column A, find the signal or signals in column B that are identical.

A	B
(1) $\delta[n+1]$	(a) $\sum_{k=-\infty}^{n} \delta[k]$
(2) $(\frac{1}{2})^n u[n]$	(b) $\frac{du(t)}{dt}$
(3) $\delta(t)$	(c) $\sum_{k=0}^{n} \delta[k]$
(4) $u(t)$	(d) $\sum_{k=0}^{\infty} (\frac{1}{2})^k \delta[n-k]$
(5) $u[n]$	(e) $\int_{-\infty}^{t} \delta(\tau)\, d\tau$
(6) $\delta[n+1]u[n]$	(f) $u[n]$
	(g) $\sum_{k=-\infty}^{\infty} (\frac{1}{2})^k \delta[n-k]$
	(h) $\delta[n+1]$
	(i) ϕ

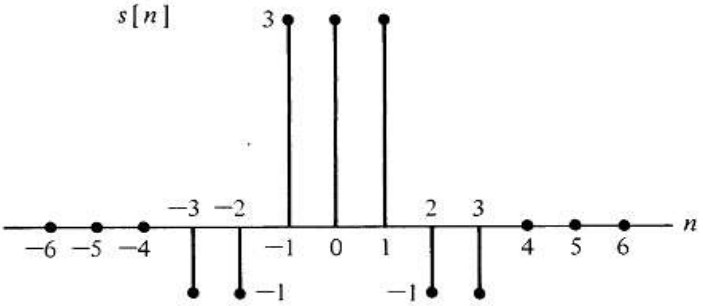
98. Express the following as sums of weighted delayed impulses, i.e., in the form

$$x[n] = \sum_{k=-\infty}^{\infty} a_k \delta[n-k]$$



99. Express the following sequence as a sum of step functions, i.e., in the form

$$s[n] = \sum_{k=-\infty}^{\infty} a_k u[n-k]$$



100. Consider an integrator that has the input-output relation

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

Determine the input-output relation for the inverse system.

101. The first-order difference equation $y[n] - a\,y[n-1] = x[n],\ 0 < a < 1$ describes a particular discrete-time system initially at rest.

i. Verify that the impulse response h[n] for this system is h[n] = a^n u[n]

ii. Is the system

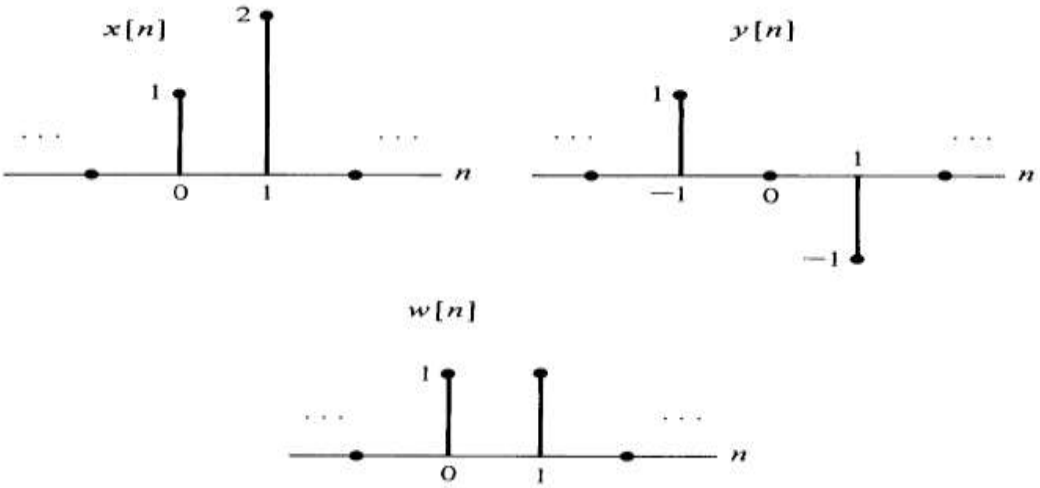
a. memoryless?

b. causal?

c. stable?

Clearly state your reasoning.

102. Consider the three discrete-time signals as shown in below Figure.



i. Verify the distributive law of convolution:

$$(x + w) * y = (x * y) + (w * y)$$

ii. You may have noticed a similarity between the convolution operation and multiplication, but they are not equivalent. Verify that

$$(x * y) \cdot w \neq x * (y \cdot w)$$

103. Determine if following statement concerning LTI systems is true or false. Justify your answer.

If h(t) is the impulse response of an LTI system and h(t) is periodic and non- zero, the system is unstable.

104. Determine if following statement concerning LTI systems is true or false. Justify your answer.

The inverse of a causal LTI system is always causal.

105. Determine if following statement concerning LTI systems is true or false. Justify your answer.

If $|h(n)| \leq$ K for each n, where K is a given number, then the LTI system with h[n] as its impulse response is stable.

106. Determine if following statement concerning LTI systems is true or false. Justify your answer.

If a discrete-time LTI system has an impulse response h[n] of finite duration, the system is stable.

107. Determine if following statement concerning LTI systems is true or false. Justify your answer.

If an LTI system is causal, it is stable.

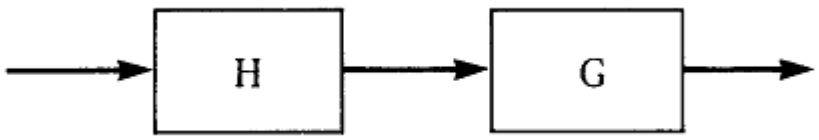
108. Determine if following statement concerning LTI systems is true or false. Justify your answer.

The cascade of a non-causal LTI system with a causal one is necessarily Non-causal.

109. Determine if following statement concerning LTI systems is true or false. Justify your answer.

A discrete-time LTI system is causal if and only if its step response s[n] is zero for n < 0.

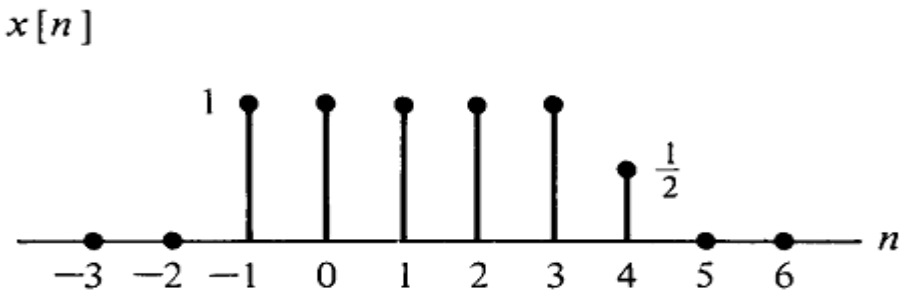
110. Consider the cascade of two systems H and G as shown in below Figure



i. If H and G are both LTI causal systems, prove that the overall system is causal.

ii. If H and G are both stable systems, show that the overall system is stable.

111. A discrete-time signal x[n] is shown in Figure



Sketch and carefully label each of the following signals:

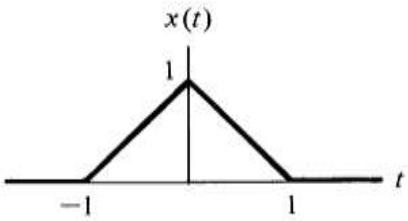
i. x[n - 2]

ii. x[4 - n]

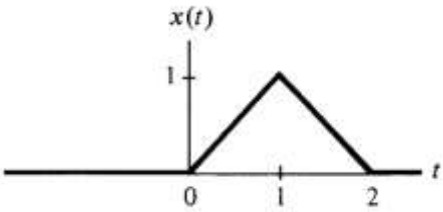
iii. x[2n]

112. For each of the following signals, determine whether it is even, odd, or neither.

i.

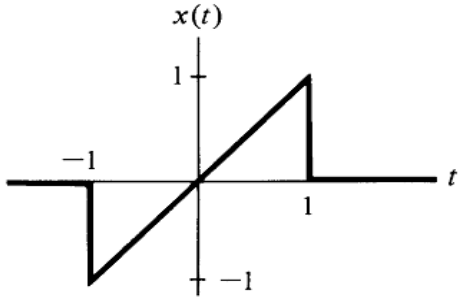


ii.

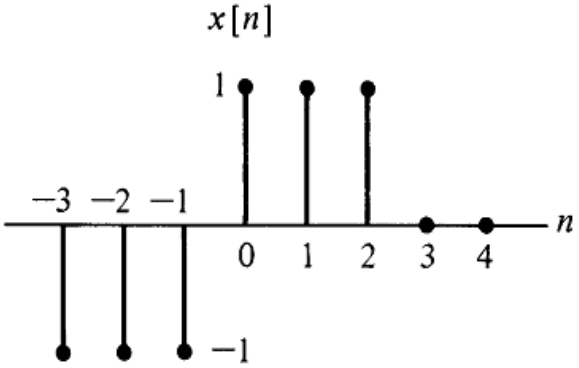


iii.

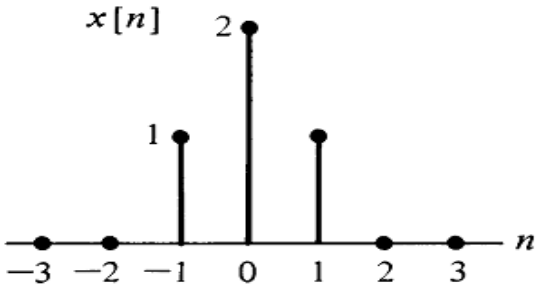
iv.



v.



vi.



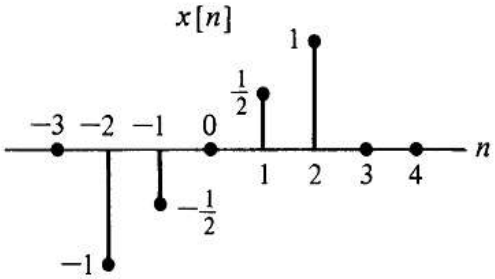
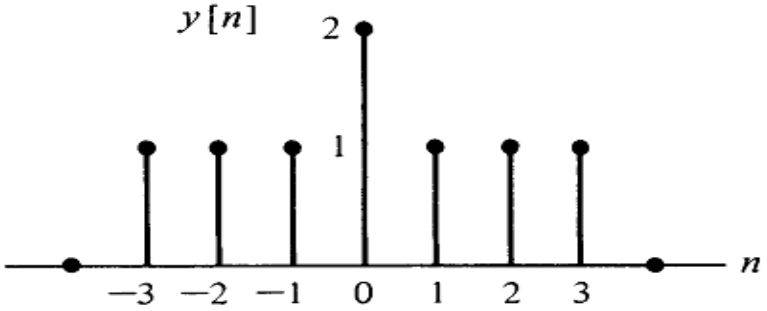


Figure (X) Figure (Y)

113. Consider the signal y[n] in Figure



$Find\ the\ signal\ x[n]\ such\ that\ Ev\{x[n]\} = y[n]\ for\ n \geq 0, and\ Od\{x[n]\} = y[n]$

for n < 0.

114. Let $x(t) = \sqrt{2}(1+j)e^{j\pi/4}e^{(-1+j2\pi)t}$

Sketch and label the following:

i. Re{x(t)}

ii. Im{x(t)}

iii. x(t + 2) + x∗(t + 2)

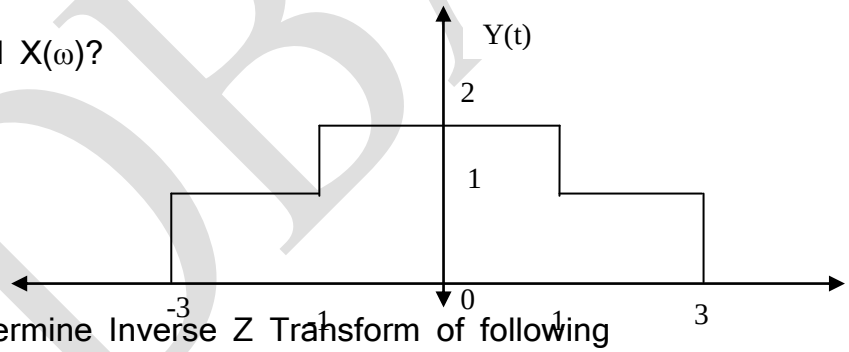
115. Evaluate the following sums:

i. $\sum_{n=0}^{5} 2\left(\frac{3}{a}\right)^n$

ii. $\sum_{n=2}^{6} b^n$

iii. $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{2n}$

116. Find X(ω)?



117. Determine Inverse Z Transform of following

i. X(Z)= $\frac{\left(1-\frac{1}{3}z^{-1}\right)}{\left(1+\frac{1}{3}z^{-1}\right)}$, X(n) is right side

ii. X(Z)= $\frac{3}{\left(z-\frac{1}{4}-\frac{1}{8}z^{-1}\right)}$, X(n) is stable

iii. X(Z)= ln (1-4z) , z < $\frac{1}{4}$

118. Find Fourier Transform of Y(t) =$\frac{4\cos(3t)}{t^2+1}$

119. Find Initial & Final Value of Transform X(Z)= $\frac{1-z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)(1+3z^{-1})}$

120. The input to the causal L.T.I. SYSTEM is X(n) = U[-n-1]+$(1/2)^n$ U(n) .

The Z. Transform of the Output is

$$Y(z) = \frac{-(1/2)Z^{-1}}{(1-(1/2)Z^{-1})(1+Z^{-1})}$$

i. Find System function with R.O.C ?

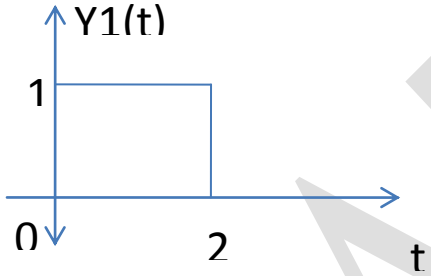
ii. What is R.O.C. of Y(z) & find Y(n)?

121. Given x(t) = 1 ; |t|<1 $\leftrightarrow$ $\frac{2\sin\omega}{\omega}$

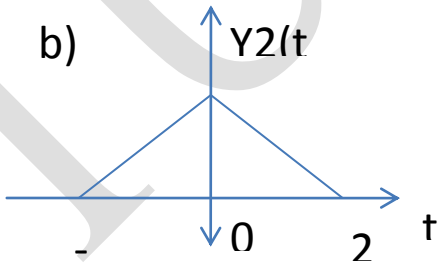
0 ; elsewhere

Find Fourier Transform of Following signals ?

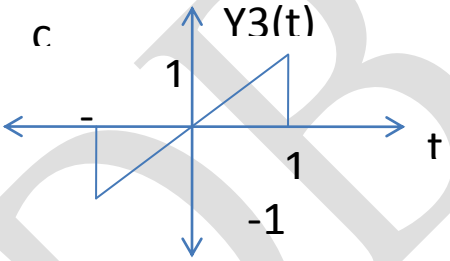
a



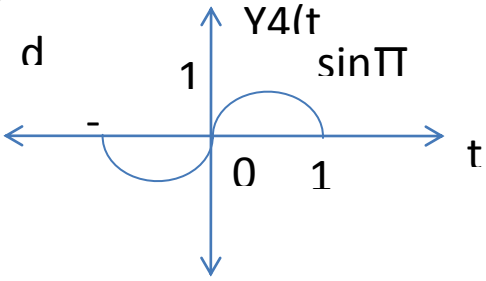
b)

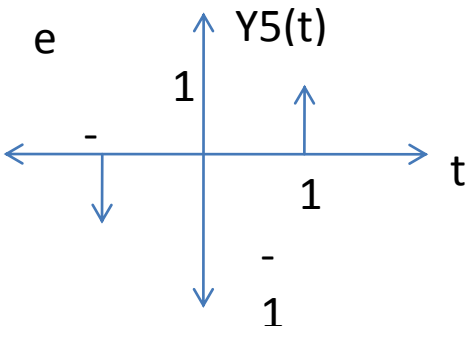


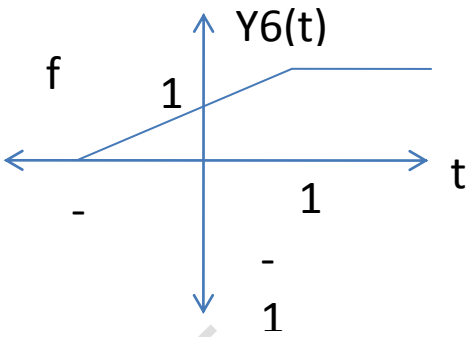
c

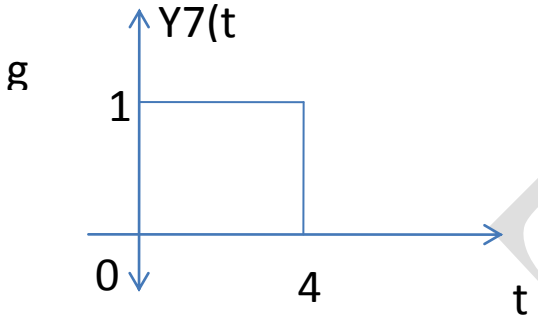


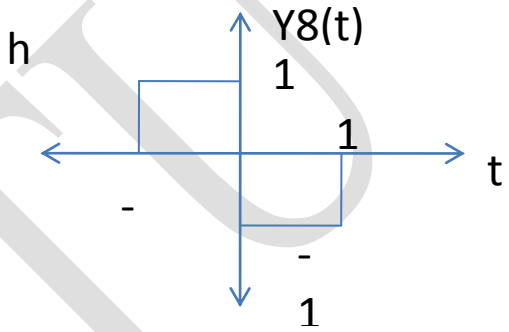
d











122. Consider the Signal $x(n) = \alpha^n u(n)$

Sketch the Signal $g(n) = x(n) - \alpha\, x(n-1)$

123. Suppose the following facts are given about the signal x(t) with laplace transform X(s)

i. x(t) is real and even

ii. X(s) has four poles and no zeros in the finite s-plane.

iii. X(s) has a pole at $s = \frac{1}{2} e^{j\pi/4}$

iv. $\int_{-\infty}^{\infty} x(t)dt = 4.$

Determine X(s) and its ROC.

124. Given that

$e^{-at}u(t) \overset{L}{\leftrightarrow} \frac{1}{s+a}$, Re{s}>a

Use properties of the laplace transform to determine the Laplace transform Y(s) of y(t).

125. In this problem we derive the multiplication property of the continuous time fourier transforn.Let x(t) and y(t) be two continuous time signals with the Fourier transforms X(jw) and Y(jw) repectively.Also let g(t) denote the inverse Fourier transform of

$$\frac{1}{2\pi}\{X(jw) * Y(jw)\}.$$

i. Show that

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)[\frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j(\omega-\theta)e^{j\omega t}d\omega]\, d\theta$$

ii. Show that

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j(\omega-\theta)e^{j\omega t}d\omega = e^{j\theta t}y(t)$$

iii. Combine the results of part i. and ii. To conclude that

$$g(t) = x(t)y(t)$$

126. Suppose we are given the following facts about an LTI system S with impulse response $h(n)$ and Frequency response $H(e^{j\omega})$:

i. $\left(\frac{1}{4}\right)^n u(n) \rightarrow g(n)$, where $g(n) = 0\ for\ n \geq 2\ and\ n < 0.$

ii. $H\left(e^{\frac{j\pi}{2}}\right) = 1$

iii. $H\left(e^{j\omega}\right) = H(e^{j(\omega-\pi)}).$

$Determine\ h(n)$

127. Let $X\left(e^{j\omega}\right)$ be the fourier transform of $x(n)$.Derive expressions in terms of $X\left(e^{j\omega}\right)$ for the fourier transform of the folowing signals.(Do not assume that $x(n)$is real)

i. Re $\{x(n)\}$

ii. $x * (-n)$

iii. Im $\{x(n)\}$

128. Determine the z-transform for the following sequences.Express all sums in closed form.Sketch the pole-zero plot and indicate the region of convergence.Indicate whether the Fourier transform of the sequence exists.

i. $\left(\frac{1}{2}\right)^n \{u[n+4] - u[n-5]\}$

ii. $n\left(\frac{1}{2}\right)^{|n|}$

iii. $|n|\left(\frac{1}{2}\right)^{|n|}$

iv. $4^n\, cos\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right)U(-n-1)$

129. Determine the sequence of the following Z-transforms by using patial fraction.

$X(z) = \frac{1-2z^{-1}}{1+\left(\frac{5}{2}\right)z^{-1}+z^{-2}}$ and x(n) is absolutely summable.

130. Determine the sequence of the following Z-transforms by using Long Division

$X(z) = \frac{(1-(\frac{1}{2})z^{-1})}{(1+(\frac{1}{2})z^{-1})}$ and x(n) is right sided.

131. Determine the sequence of the following Z-transforms by using patial fraction

$X(z) = \frac{3}{z-\frac{1}{4}-\frac{1}{8}z^{-1}}$ and x(n) is absolutely summable.

132. Consider an even sequence $x[n]\{i.e. x[n] = x[-n]\}$ with rational z-transform X(z).

i. From the definition of the z-transform, show that

$$X(z) = X\left(\frac{1}{z}\right)$$

ii. From your results in part i. show that if a pole (zero) of X(z) occurs at $z = z_0$.then a pole (zero) must also occur at $z = \frac{1}{z_0}$

iii. Verify the result in part ii. For each of the following sequences:

a) $\delta[n+1] + \delta[n-1]$

b) $\delta[n+1] - \frac{5}{2}\delta[n] + \delta[n-1]$

133. What is sampling theorem and what are its applications ?

134. For the following signals,

i. determine analytically which are periodic (if periodic, give the period) and

ii. sketch the signals. (Scale your time axis so that a sufficient amount of the signal is being plotted.).

a) $x(t) = 4\,cos(5\pi t)$

b) $x(t) = 4\,cos(5\pi t - \pi/4)$

c) $x(t) = 4u(t) + 2sin(3t)$

d) $x(t) = u(t) - ½)$

e) $x[n] = 4cos(\pi n - 2)$

f) $x[n] = 2sin(3n)$

g) $x[n] = u[n] + p_4[n]$

135. Determine if the following signals are periodic; if periodic, give the period.

   i. x(t) = cos(4t) + 2sin(8t)

   ii. x(t) = cos(3 πt) + 2cos(4 π t)

136. Find the z transform of the following signals:

   i. x[n] = u[n] - u[n-4]

   ii. x[n] = 0.5^n u[n]

137. Find the z transform of the following signals:

   i. x[n] = [1 4 8 2]

   ii. x[n] = [0 1 2 3 4]

   iii. x[n] = $2(0.8)^n$ u[n]

138. Find the inverse Z-transforms of the following signals:

   i.

$$X(z) = \frac{(z-1)(z+0.8)}{(z-0.5)(z+0.2)}$$

   ii.

$$X(z) = \frac{(z+0.8)}{(z-0.5)(z+0.2)}$$

139. Sketch the following signals

   i. $x(t) = \begin{cases} 0 & if \quad t < -4 \\ t+2 & if \quad -4 \le t < 3 \\ t-2 & if \quad 3 \le t \end{cases}$

   ii. y(t) = x(t-1) where x(t) is defined in part i.

140. Sketch the following signals

   i. $x(t) = \begin{cases} 0 & if \quad n < 2 \\ 2n-4 & if \quad 2 \le n < 4 \\ 4-n & if \quad 4 \le n \end{cases}$

   ii. y[n] = x[n+1] where x[n] is defined in part i.

141. Determine if the following systems are time-invariant, linear, causal, and/or memoryless?

   i.

$$\frac{dy}{dt} + 6y(t) = 4x(t)$$

ii.

$$\frac{dy}{dt} + 4ty(t) = 2x(t)$$

iii.

$$y[n] + 2y[n-1] = x[n+1]$$

iv.

$$y(t) = \sin(x(t))$$

142. Determine if the following systems are time-invariant, linear, causal, and/or memoryless?

i.

$$\frac{dy}{dt} + y^2(t) = x(t)$$

ii.

$$y[n+1] + 4y[n] = 3x[n+1] - x[n]$$

iii.

$$y[n] = x[2n]$$

143. Determine if the following systems are time-invariant, linear, causal, and/or memoryless?

i.

$$y[n] = nx[2n]$$

ii.

$$\frac{dy}{dt} + \sin(t)y(t) = 4x(t)$$

iii.

$$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 4y(t) = \frac{dx}{dt} + 4x(t)$$

144. The response of an LTI system to a step input, x(t) = u(t) is y(t) = $(1-e^{-2t})$ u(t). What is the response to an input of x(t) = 4u(t)-4u(t-1)?

145. Find the impulse response for each of the following discrete-time systems.

   i. y[n] + 0.2y[n-1] = x[n]-x[n-1]

   ii. y[n] + 1.2y[n-1] = 2x[n-1]

   iii. y[n] = 0.24(x[n]+x[n-1]+x[n-2]+x[n-3])

   iv. y[n] = x[n] + 0.5x[n-1] + x[n-2]

146. Perform the following convolutions, x[n]∗v[n]

   i. x[n] = u[n] - u[n-4], v[n] = 0.5^nu[n]

   ii. x[n] = [1 4 8 2]; v[n] = [0 1 2 3 4] (the sequences both start at n=0)

   iii. x[n] = u[n], v[n] = $2(0.8)^n$ u[n]

   iv. x[n] = u[n - 1], v[n] = $2(0.5)^n$ u[n]

147. **State Sampling theorem.**
148. **What is meant by aliasing?**
149. **What are the effects aliasing?**
150. **How the aliasing process is eliminated.**
151. **Define Nyquist rate.and Nyquist interval.**
152. **Define sampling of band pass signals.**
153. **Define Z transform.**
154. **Define unilateral Z transform.**
155. **What is region of Convergence.**
156. **What are the Properties of ROC.**
157. **What is the relationship between Z transform and fourier transform.**
158. **What is meant by step response of the DT system.**
159. **Define Transfer function of the DT system.**
160. **Define frequency response of the DT system.**
161. **What is the condition for stable system.**
162. **What are the blocks used for block diagram representation.**
163. **State the significance of block diagram representation.**

164. **State the Commutative properties of convolution.**
165. **State the Associative properties of convolution.**
166. **State Distributive properties of convolution.**
167. **Check whether the system is causal or not ,the H(z) is given by (z^3 + z)/(z+1).**
168. **Check whether the system is stable or not ,the H(z) is given by (z/z-a)., lal < 1.**

169. **Determine the z-transform of following sequences?**
    i. **Unit Impulse Response**
    ii. **Unit step response**
170. **An LTI system is described by the difference equation y[n] −2y[n−1] =x[n−1]−x[n−2]. Compute the output y[n] if the input x[n] = 3^n u[n].**

171. **Find the z transforms of** $x[n] = e^{-\frac{n}{40}} u[n]$ **and** $x_m[n] = e^{-\frac{n}{40}} \sin\left(\frac{2\pi n}{8}\right) u[n]$
172. Using the time shifting properties, find the z-transform of these signals
    i. $x[n] = u[n-5]$
    ii. $x[n] = u[n+2]$
    iii. $x[n] = (\frac{2}{3})^n u[n+2]$
173. **Using the convolution property, find the z-transform of these signals.**

$$x[n] = (0.9)^n u[n] * u[n]$$

174. **Using the convolution property, find the z-transform of these signals**

$$x[n] = (0.9)^n u[n] * (0.6)^n u[n]$$