Answer Book
of
DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY,
LONERE – RAIGAD -402 103
End Semester Examination – December - 2017

Branch: B. Tech (Group A/Group B) **Sem.:- I**

Subject with Subject Code:-Engineering Mechanics ME 102 **Marks: 60**

Date:- 13/12/2017 **Time:- 3 Hr.**

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Instructions to the Students

1. Each question carries 12 marks.
2. Attempt any five questions of the following.
3. Illustrate your answers with neat sketches, diagram etc., wherever necessary.
4. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly

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(Marks)

Q.1. A) How will you add two forces? Explain the Parallelogram Law and Law of Triangle of forces. **(6)**

2.3 ADDITION OF TWO FORCES : PARALLELOGRAM LAW

Before discussing the parallelogram law, let us clarify the concept of *equal forces* and *equivalent forces*.

Two forces are said to be *equal* if they have the same magnitude and direction even if their points of application are not the same.

Two forces are said to be *equivalent* if (in some sense) they produce the some effect on a rigid body. To clarify the point let us consider two 25 paise coins and one 50 paise coin. They are not equal in size, shape and weight yet are equivalent in their buying capacity. Interestingly, they are not equivalent in their capacity to operate a 50 paise coin operated public telephone. *Equivalence is thus based on some specific effect.*

Most of the time in mechanics, we are concerned with the forces having an equivalent effect on a rigid body rather than the equal forces. *The resultant of two forces (or their sum) acting on a body, in this sense, is a equivalent force*

Parallelogram Law. It was mentioned earlier that two forces add according to the parallelogram law. This law can be stated as "*If two force*

* Vector quantities have not been distinguished by bold faced letter in the solved examples of this book.

acting at a point are represented in magnitude and direction by the adjacent sides of a parallelogram then the diagonal passing through their point of intersection represents the resultant in both magnitude and direction."

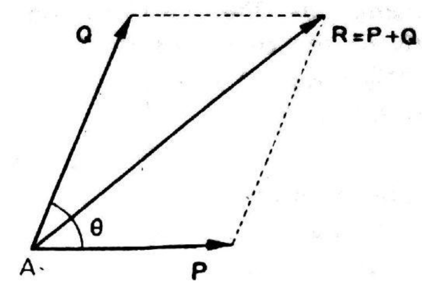


Fig. 2.2

The sum of the two forces **P** and **Q** acting at the point *A*, with the included angle θ, can be obtained by constructing a parallelogram such that the forces **P** and **Q** represent the two adjacent sides of the parallelogram as shown in Fig. 2.2.

The diagonal that passes through the point *A* represents the sum or the resultant **R** of the forces **P** and **Q**

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} \qquad ...(2.1)$$

The sum or the resultant of **P** and **Q** is independent of the order in which they are added.

$$\mathbf{P} + \mathbf{Q} = \mathbf{Q} + \mathbf{P} \qquad ...(2.2)$$

Law of Triangle of Forces. Instead of constructing the parallelogram the sum of the resultant of the two forces can be determined by the triangle law.

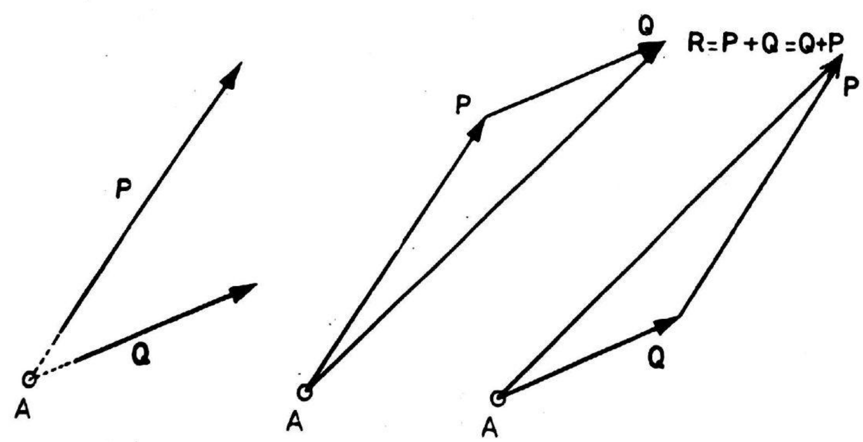


Fig. 2.3

This can be stated as "*If two forces acting at a point are represented by two sides of a triangle taken in order, then their sum or resultant is represented by the third side taken in an opposite order.*"

The sum or resultant of two forces **P** and **Q** acting at point *A* [Fig. 2.3 (*a*)] can be obtained by constructing a triangle such that the forces **P** and **Q** are represented by the two sides of this triangle taken in an order. The closing side [Fig. 2.3 (*b*)] taken in an opposite order then ... sum or the resultant **R** of the forces **P** and **Q**.

The forces **P** and **Q** can be added in any order as shown in Fig... and (*c*) as,

$$\mathbf{P} + \mathbf{Q} = \mathbf{Q} + \mathbf{P}$$

The magnitude of the resultant **R** can be determined graphic... measuring the length of vector **R** of the force triangle.

The magnitude of the resultant **R** can also be determined trigonome... if the included angle β between the forces **P** and **Q** is known (Fig. 2.4) ... the relation

$$R^2 = P^2 + Q^2 - 2\,PQ \cos \beta$$

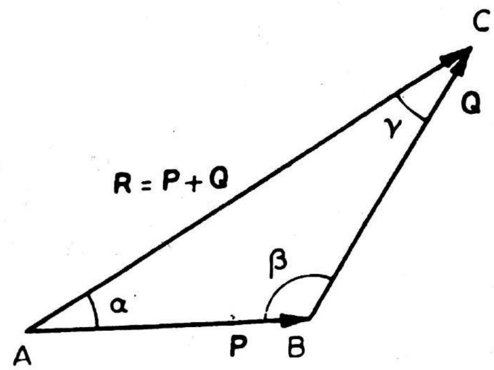


Fig. 2.4

The remaining angles can be computed using the law of sines as,

$$\frac{P}{\sin \gamma} = \frac{Q}{\sin \alpha} = \frac{R}{\sin \beta} \qquad ...(2.$$

B) Two ropes are tied together at C. If the maximum permissible tension in each rope is 3.5 kN, what is the maximum force P that can be applied and in what direction as shown in figure 1. (6)

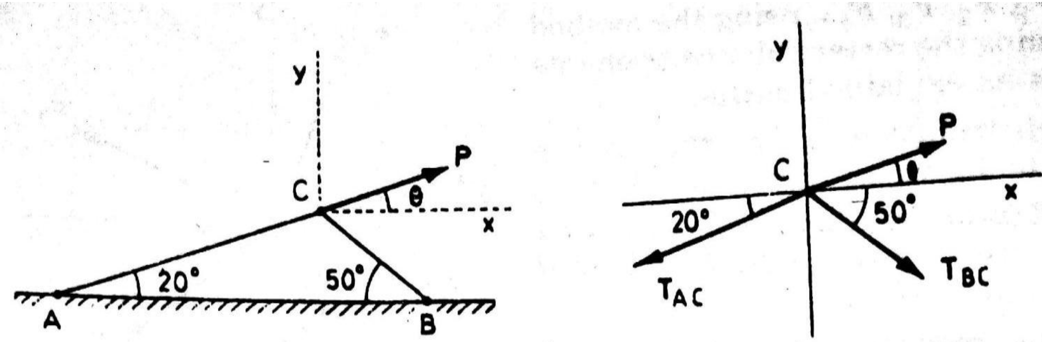


Fig. 2.18

Solution. Consider the equilibrium of the point C,

$\Sigma F_x = 0$: $P\cos\theta + T_{BC}\cos 50° - T_{AC}\cos 20° = 0$

$\Sigma F_y = 0$: $P\sin\theta - T_{BC}\sin 50° - T_{AC}\sin 20° = 0$

where, T_{AC} and T_{BC} are tension in the strings AC and BC

$$P\cos\theta = T_{AC}\cos 20° - T_{BC}\cos 50° \quad ...(i)$$

$$P\sin\theta = T_{AC}\sin 20° + T_{BC}\sin 50° \quad ...(ii)$$

Dividing (*ii*) by (*i*)

$$\frac{P\sin\theta}{P\cos\theta} = \tan\theta = \frac{T_{AC}\sin 20° + T_{BC}\sin 50°}{T_{AC}\cos 20° - T_{BC}\cos 50°}$$

But $T_{AC} = T_{BC} = 3.5 \text{ kN}$

$$\tan\theta = \frac{0.766 + 0.342}{0.94 - 0.643} = 3.703$$

$$\theta = 75° \ \textit{Ans.}$$

Substituting $\theta = 75°$ and $T_{AC} = T_{BC} = 3.5$ kN in (*i*)

$$P = \frac{3.5(0.94 - 0.643)}{\cos 75°} = \frac{1.04}{0.259} = 4.0 \text{ kN}$$

$P = 4.0$ kN *Ans.*

Q.2. A) What are various types of supports and support reactions? Explain with its free body diagram. (4)

2.10 TYPES OF SUPPORTS AND SUPPORT REACTIONS

1. Frictionless Support. The reaction acts normal to the surface at the point of contact as shown in Fig. 2.19.

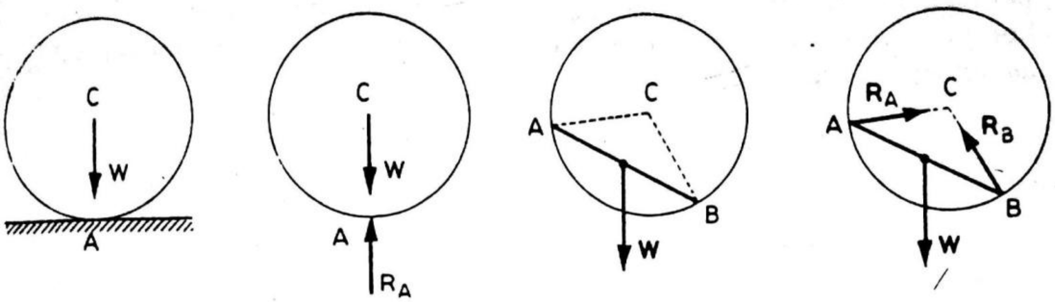


Fig. 2.19

2. Roller and Knife Edge Supports. The roller and the knife edge restrict the motion normal to the surface of the beam *AB*. So, reactions R_A and R_B shall act normal to the surface at the points of contact *A* and *B* as shown in Fig. 2.20.

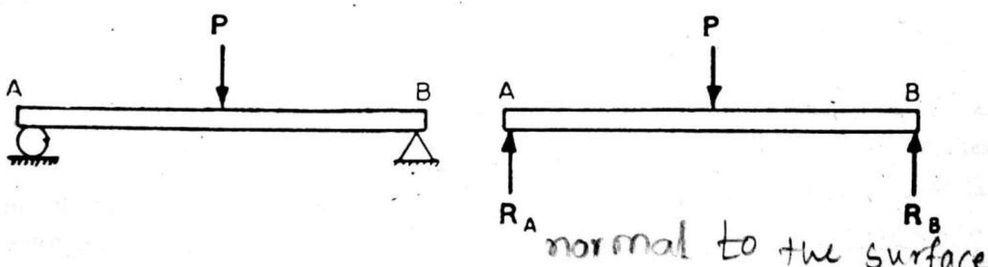


Fig. 2.20

3. Hinged Support. The hinge restricts the motion of the end *A* of the beam *AB* both in the horizontal as well as vertical directions. Thus there are two *independent* reactions X_A and Y_A acting on the beam at *A*.

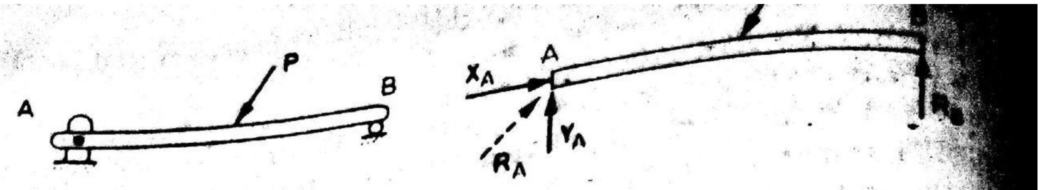


Fig. 2.21

These two rectangular components can be combined into a single force or reaction R_A. Therefore, *the reaction at the hinge can be represented by a single force* R_A *in an unknown direction or by the components* X_A and Y_A. The reaction at a hinge whether represented by a single force or by its two rectangular components, involves two unknowns (one direction one magnitude or two magnitudes). This is shown in Fig. 2.21.

4. Built-in-Support. If the end *A* of a beam *AB* is embedded in the concrete, it restricts the motion of the end *A in* the horizontal and the vertical directions. It also restricts the rotation of the beam *AB* about the point *A*. The reactions X_A and Y_A, therefore, shall be exerted both in the horizontal and the vertical directions accompanied by a reaction couple M_A as shown in Fig. 2.22.

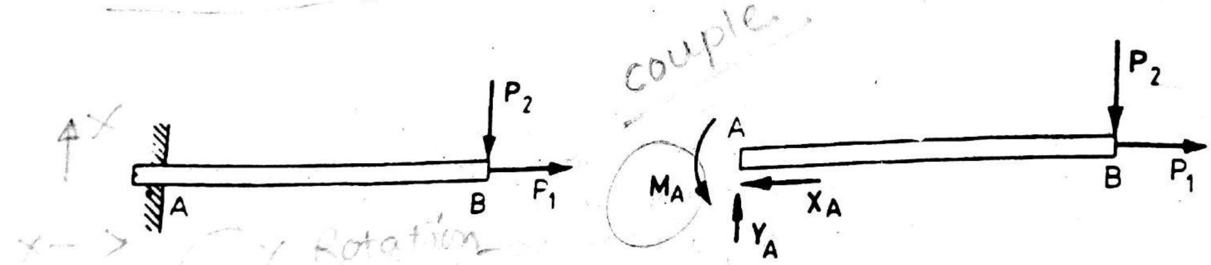


Fig. 2.22

2.11 FREE-BODY DIAGRAM

To clearly identify the various forces acting on a body in equilibrium we have to draw its free-body diagram. Only then, we can write the equations of the equilibrium of the body. This concept should be thoroughly mastered before attempting any further study of the subject.

To draw the free-body diagram of a body we remove all the supports (like wall, floor, hinge or any other body) and replace them by the reactions which these supports exert on the body.

Before discussing this concept of free-body diagram further, let us consider two types of forces that act on a body. They are : *external forces* and *internal forces.*

External Forces. These are forces which act on a body or a system of bodies from outside. For example, in the case of the roller shown in the Fig. 2.23, (*i*) Weight of roller **W**, (*ii*) Applied force **P** and (*iii*) The reaction

B) Using the method of joints, find the axial forces in all the members of a truss with the loading shown in the Figure 2. (8)

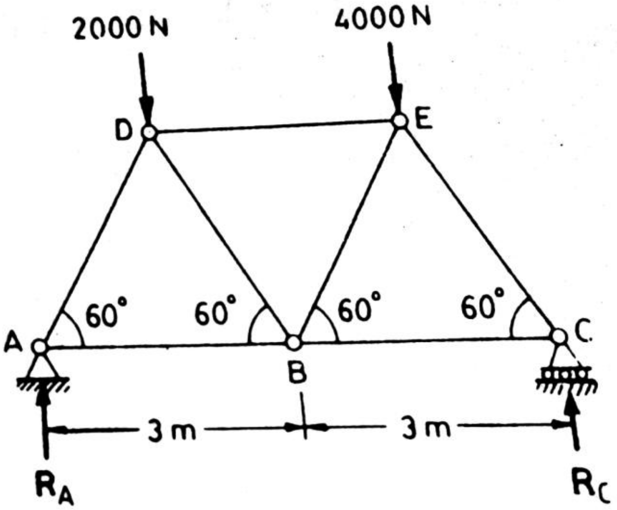


Figure 2

Fig. 9.7

Solution. **Entire Truss.** To determine the support reactions consider the equilibrium of the entire truss.

In general, the reaction at a hinge can have two components acting in the horizontal and the vertical directions. As there is no horizontal external force acting on the truss, so the horizontal component of reaction at *A* is zero.

Taking moments about *A*,

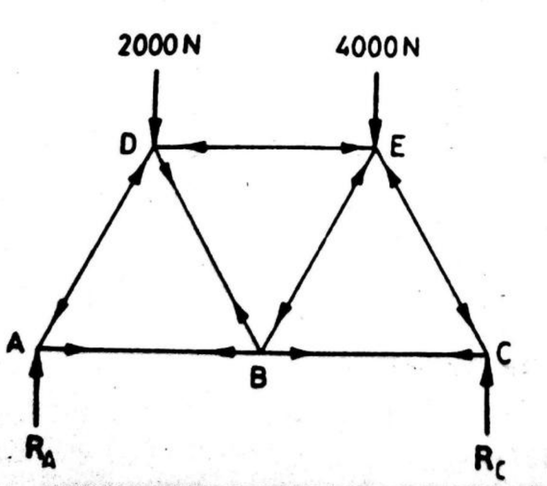
$$\Sigma M_A = 0: \qquad -2000 \times (1.5) - 4000(4.5) + R_C \times (6) = 0$$

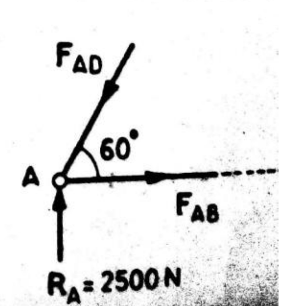
$$R_C = 3500 \text{ N}$$

$$\Sigma F_y = 0: \qquad R_A + R_C - 2000 - 4000 = 0$$

$$R_A = 2500 \text{ N}$$

Before considering the equilibrium of the joints, mark by inspection, the directions of axial forces in all the members as shown in Fig. 9.8.





Joint A. Let us begin with the joint A at which there ... unknown forces. We cannot begin with the joint D because, th... unknown forces acting at the joint D.

Consider the free-body diagram of the joint A. Equations of ... can be written as

$$\Sigma F_x = 0: \quad F_{AB} - F_{AD} \cos 60° = 0$$

$$\Sigma F_y = 0: \quad R_A - F_{AD} \sin 60° = 0$$

$$F_{AD} = \frac{R_A}{\sin 60°} = \frac{2500}{0.866}$$

$$F_{AD} = 2887 \text{ N(C)} \qquad \textit{Ans.}$$

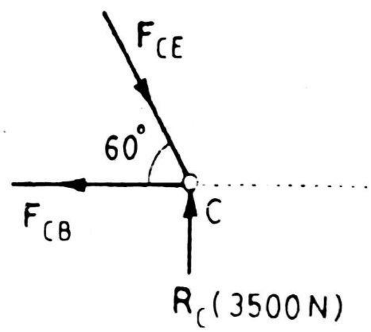
Using (i)

$$F_{AB} = F_{AD} \cos 60° = 2887 \times 0.5$$

$$F_{AB} = 1443 \text{ N(T)} \qquad \textit{Ans.}$$

The magnitudes of the forces F_{AB} and F_{AD} are both coming positive, therefore, the assumed direction of the forces are corr...

Joint C



Joint C

$$\Sigma F_x = 0: \quad R_{CE} \cos 60° - F_{CB} = 0$$

$$\Sigma F_y = 0: \quad R_C - R_{CE} \sin 60° = 0$$

$$F_{CE} = \frac{R_C}{\sin 60°} = \frac{3500}{0.866}$$

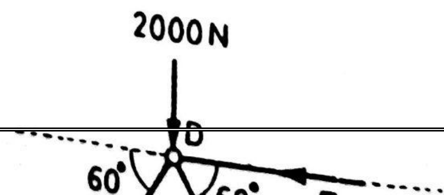
$$F_{CE} = 4041 \text{ N(C)} \quad \textit{Ans.}$$

From (iii)

$$F_{CB} = \frac{F_{CE}}{\cos 60°} = \frac{4041}{0.5}$$

Joint D

$$F_{CB} = 2020.5 \text{ N (T)} \quad \textit{Ans.}$$



$$F_{AD} = 2887 \text{ N (known)}$$

$\Sigma F_x = 0:$ $\quad F_{DB} \cos 60° + F_{AD} \cos 60° - F_{DE} = 0 \quad$ ...(v)

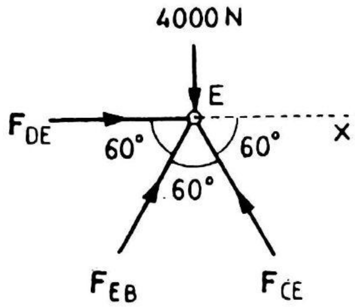
$\Sigma F_y = 0:$ $\quad F_{AD} \sin 60° - F_{DB} \sin 60° - 2000 = 0 \quad$ ...(vi)

$$F_{DB} = \frac{2887 \times 0.866 - 2000}{0.866}$$

$$F_{DB} = 577 \text{ N(T)} \quad \textit{Ans.}$$

From (v)
$$F_{DE} = F_{DB} \cos 60° + F_{AD} \cos 60°$$
$$F_{DE} = 577 \times 0.5 + 2887 \times 0.5$$
$$F_{DE} = 1732 \text{ N(C)} \quad \textit{Ans.}$$

Joint E



Joint E

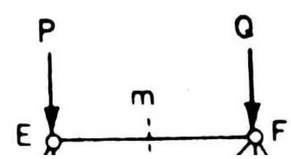
$$F_{DE} = 1732 \text{ N}$$
$$F_{CE} = 4041 \text{ N (known)}$$

$\Sigma F_x = 0:$ $\quad F_{DE} + F_{EB} \cos 60° - F_{CE} \cos 60° = 0 \quad$ ...(vii)

Or
$$F_{EB} \cos 60° = F_{CE} \cos 60° - F_{DE}$$

Or
$$F_{EB} = \frac{4041 \times 0.5 - 1732}{0.5}$$
$$F_{EB} = 577 \text{ N(C)} \quad \textit{Ans.}$$

There is no need to consider the equilibrium of the joint B as all the forces have been determined.



Q.3. A) Locate the centroid of the shaded area obtained by removing a semicircle of diameter *a* from a quadrant of a circle of radius *a* as shown in Fig. 3. **(6)**

of diameter a from a quadrant of a circle of radius a.

Solution. To determine the centroid (x_c, y_c) of the shaded area let us consider that the shaded area is obtained by subtracting the area of the semicircle of radius $a/2$ from the area of the quadrant of circle of radius a. In this sense, therefore, the area to be subtracted is treated to be as negative area.

Reference axes are as shown in the Fig. 4.16. Different areas and coordinates of their centroids are tabulated below :

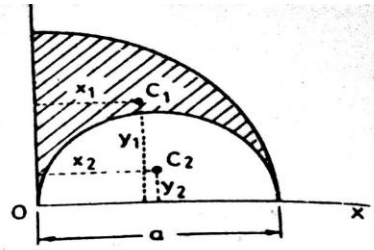


Fig. 4.16

Figure	*Area*	*x-coordinate of the centroid*	*y-coordinate of the centroid*
Quadrant of circle of radius = a	$A_1 = \frac{\pi}{4}a^2$	$x_1 = \frac{4a}{3\pi}$	$y_1 = \frac{4a}{3\pi}$
Semicircle of radius = $a/2$	$A = \frac{\pi(a/2)^2}{2}$ $A_2 = -\frac{\pi a^2}{8}$	$x_2 = \frac{a}{2}$	$y_2 = \frac{4}{3\pi}\left(\frac{a}{2}\right) = \frac{2a}{3\pi}$

Centroid of the composite area

$$x_c = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$x_c = \frac{\frac{\pi}{4}a^2\left(\frac{4a}{3\pi}\right) - \frac{\pi a^2}{8}\left(\frac{a}{2}\right)}{\frac{\pi a^2}{4} - \frac{\pi a^2}{8}}$$

$$x_c = \frac{\frac{a}{3\pi} - \frac{a}{16}}{\frac{1}{8}} = 8a\left(\frac{1}{3\pi} - \frac{1}{16}\right)$$

$x_c = 0.349a$ *Ans.*

$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$y_c = \frac{\frac{\pi}{4}a^2\left(\frac{4a}{3\pi}\right) - \frac{\pi a^2}{8}\left(\frac{2a}{3\pi}\right)}{\frac{\pi a^2}{4} - \frac{\pi a^2}{8}}$$

$$y_c = \frac{\frac{a}{3\pi} - \frac{a}{12\pi}}{\frac{1}{8}} = \frac{2a}{\pi}$$

$y_c = 0.636a$ *Ans.*

B) A 7 m long ladder rests against a vertical wall, with which it makes an angle of 45°, and on a floor. If a man, whose weight is one half of that of the ladder, climbs it, at what distance along the ladder will he be, when the ladder is about to slip shown in Figure 4? The coefficient of friction between the ladder and the wall is 1/3 and that between the ladder and the floor is 1/2. (6)

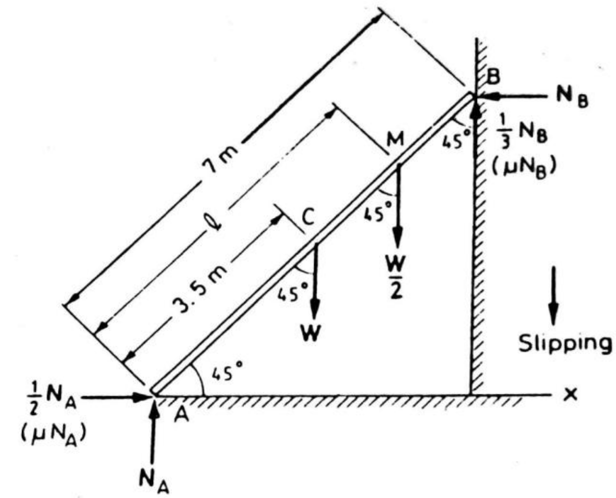


Fig. 6.8

Solution. Suppose the man climbs a length l of the ladder before slippi impends. Free-body diagram is as shown in Fig. 6.8.

Writing the equations of equilibrium,

$\Sigma F_x = 0:$ $$\frac{1}{2}N_A - N_B = 0 \quad ...$$

$\Sigma F_y = 0:$ $$N_A + \frac{N_B}{3} - W - \frac{W}{2} = 0 \quad ...(ii)$$

Taking moments about A,

$\Sigma M_A = 0:$

$$N_B(7\sin 45°) + \frac{1}{3}N_B(7\cos 45°) - W\left(\frac{7}{2}\cos 45°\right) - \frac{W}{2}(l\cos 45°) = 0 \quad ...(iii)$$

From (*i*) $N_A = 2N_B$, substituting in (*ii*)

$$2N_B + \frac{N_B}{3} - \frac{3W}{2} = 0$$

$$N_B = \frac{9}{14}W,\ N_A = \frac{9}{7}W$$

Substituting for N_A and N_B in (*iii*)

$$\frac{9}{14}W(7\sin 45°) + \frac{1}{3}\left(\frac{9}{14}W\right)(7\cos 45°) - W\left(\frac{7}{2}\cos 45°\right) - \frac{W}{2}(l\cos 45°) = 0$$

$$\frac{9}{14}\times 7 + \frac{1}{3}.\frac{9}{14}\times 7 - \frac{7}{2} = \frac{l}{2}$$

$$l = 5\text{ m} \quad \textit{Ans.}$$

Q.4. A) A trolley resting on a horizontal plane starts from rest and is moved to the right with a constant acceleration of 0.18 m/s^2 shown in Figure 5. Determine i) acceleration of the block B connected to the trolley and ii) velocities of the trolley and the block after a time of 4 seconds and the distance moved by each of them. (6)

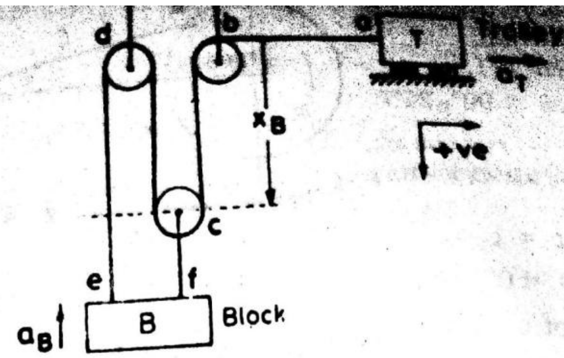


Fig. 14.9

$$x_T + 3x_B + cf = \text{constant}$$

It may be noted that the motions of the centre of the pulley c and B are identical, hence expressed in terms of x_B. Otherwise also, di cf remains constant.

Giving an increment Δx_T to the trolley

$$\Delta x_T + 3\Delta X_B = 0$$

Differentiating equation (*i*) w.r.t. time

$$\frac{dx_T}{dt} + 3\frac{dx_B}{dt} = 0 \quad \text{or} \quad v_T + 3v_B = 0$$

Differentiating again,

$$\frac{d^2x_T}{dt^2} + 3\frac{d^2x_B}{dt^2} = 0 \quad \text{or} \quad a_T + 3a_B = 0$$

Acceleration of the block is given by the equation (*iv*), given

$$a_T = \overrightarrow{0.18} \text{ m/s}^2$$

$$a_B = -\frac{a_T}{3} = \frac{0.18}{3} = 0.06 \overset{\uparrow}{\text{m}}/\text{s}^2 \quad \textit{Ans.}$$

(–ve sign indicates direction)

Velocity of the trolley 4 seconds after starting from rest is

$$v = u + at$$

$$v_T = 0 + 0.18 \times 4 = 0.72 \text{ m/s} \quad \textit{Ans.}$$

Velocity the block is given by the equation (*iii*)

$$v_B = -\frac{v_T}{3} = \frac{0.72}{3} = 0.24 \overset{\uparrow}{\text{m/s}} \quad Ans.$$

Distance moved by the trolley in 4 seconds is given by,

$$S = ut + \frac{1}{2}at^2$$

$$S_T = 0 + \frac{1}{2}(0.18)\,(4)^2$$

$$S_T = 1.44 \text{ m} \quad Ans.$$

Distance moved by the block is given by the equation (*ii*)

$$S_B = -\frac{S_T}{3} = \frac{1.44}{3} = 0.48 \text{ m} \quad Ans.$$

B) A passenger train passes a certain station at 60 km/hr and covers a distance of 12 km with this speed and then stops at the next station 15 km from the first with uniform retardation. A local train starting from the first station covers the same distance in double this time and stops at the next station. Determine the maximum speed of the local train which covers a part of the distance with uniform acceleration and the rest with uniform retardation.

of the distance ... retardation.

Solution. The motions of the two trains are shown on veloc... (Fig. 14.15).

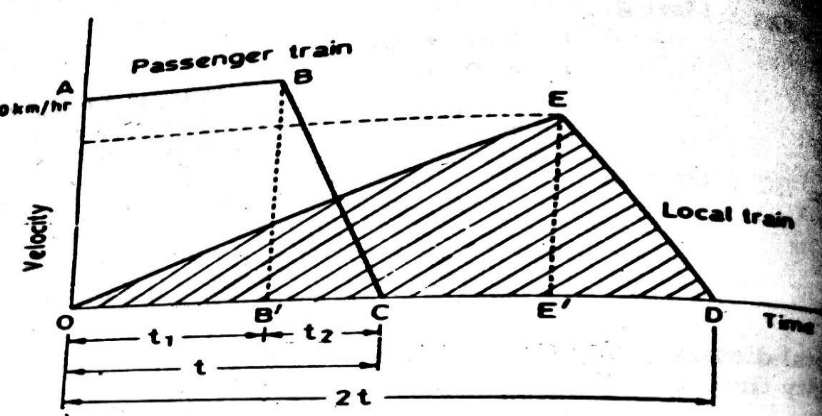


Fig. 14.15

Passenger Train : It moves out from a certain station with a speed of 60 km/hr (*AB*) and after travelling a distance of 12 km re... stops at the next station (*BC*).

Area under the velocity-time graph represents the distance tr... the train.

$$= \text{Area } OABC$$

$$\text{Area } OABC = \text{Area } OABB' + \text{Area } BB'C$$

$$\text{Area } OABB' = 12 \text{ km} = v \times t_1 = 60 \times t_1$$

$$t_1 = \frac{12}{60} = \frac{1}{5} \text{ hr}$$

$$\text{Area } BB'C = \frac{vt_2}{2} = 3 \text{ km}$$

$$\frac{60 \times t_2}{2} = 3$$

$$t_2 = \frac{3 \times 2}{60} = \frac{1}{10} \text{ hr}$$

Total time of travel of the passenger train,

$$t = t_1 + t_2 = \frac{1}{5} + \frac{1}{10} = \frac{3}{10} \text{hr}$$

Local Train :

Time of travel of the local train = 2 × time of passenger train

$$= 2 \times \frac{3}{10} = \frac{3}{5} \text{hr}$$

Distance travelled by local train = 15 km

$$\text{Area} \quad OED = \frac{V_{max}}{2} \times 2t$$

$$15 = V_{max} \times t$$

$$15 = V_{max} \times \left(\frac{3}{10}\right)$$

$$V_{max} = \frac{15 \times 10}{3}, \; V_{max} = 50 \text{ km/hr} \quad \textit{Ans.}$$

Q.5. A) Explain in detail: D'Alemberts principle and write the equations of dynamic equilibrium of the particle. (4)

14.7 EQUATIONS OF DYNAMIC EQUILIBRIUM: D'ALEMBERT'S PRINCIPLE

The equation of motion of the particle P

$$\Sigma \mathbf{F} = m\mathbf{a}$$

can be written in the form

$$\Sigma \mathbf{F} - m\mathbf{a} = 0$$

which means that the resultant of the external forces ($\Sigma \mathbf{F}$) and the ($-m\mathbf{a}$) is zero. The force ($-m\mathbf{a}$) is called inertia force. The inertia of a body can be defined as the resistance to the change in the condition of rest or of uniform motion of a body.

The magnitude of the inertia force is equal to the product of the mass and acceleration of the particle and it acts in a direction opposite to the direction of acceleration of the particle.

The equations in the form

$$\Sigma \mathbf{F} - m\mathbf{a} = 0$$

Or

$$\Sigma \mathbf{F} + \underbrace{(-m\mathbf{a})}_{\text{Inertia Force}} = 0 \qquad ...(14.18)$$

or in the component form

$$\Sigma F_x + \underbrace{(-ma_x)}_{\text{Inertia Force}} = 0 \qquad ...(14.19)$$

$$\Sigma F_y + \underbrace{(-ma_y)}_{\text{Inertia Force}} = 0 \qquad ...(14.20)$$

are called *the equations of dynamic equilibrium of the particle.*

So, to write the equation of dynamic equilibrium of a particle add a fictitious force equal to the inertia force to the external forces acting on the particle and equate the sum (resultant) to zero (Fig. 14.19). This concept is known as D'Alembert's Principle. It is very useful concept as moment equation of dynamic equilibrium is easy to conceive and write.

For the simplicity of representation, equations (14.19) and (14.20) can be written as

$$\Sigma F_x = 0 \qquad ...(14.21)$$

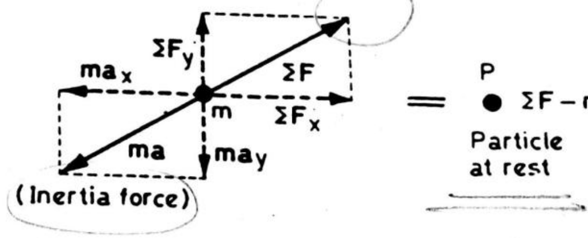
$$\Sigma F_y = 0 \qquad ...(14.22)$$

When expressed in the above manner, the terms ΣF_x and ΣF_y are to be *redefined*. That is, ΣF_x and ΣF_y now include the inertia forces also.

Further the equations 14.21 and 14.22 have an appearance similar to the equations of static equilibrium.

It should be clearly understood that the equation of motion of a particle and the equation of dynamic equilibrium of a particle are the two methods of expression which differ only in the concept used and in the manner of writing the equations. The final result, however, shall be the same.

Whichever method is followed should be followed consistently and clearly.



B) Two blocks of masses M1 and M2 are connected by a flexible but inextensible string as shown in the figure. Assuming the coefficient of friction between block M1 and horizontal surface to be μ find the acceleration of the masses and tension in the string as per figure 6. Assume M1= 10 kg and M2 = 5 kg, μ = 0.25. (8)

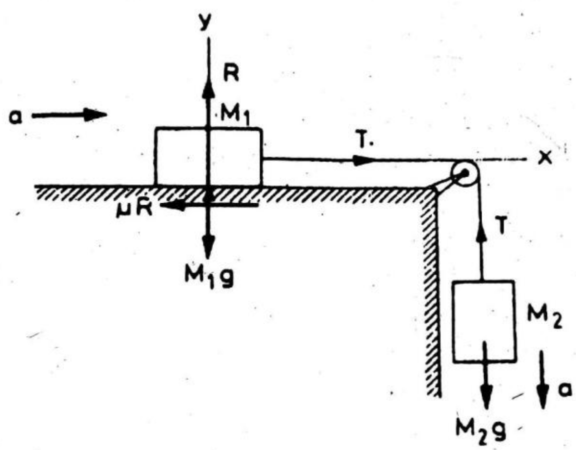


Fig. 14.23

Solution. Let the mass M_2 move down with an acceleration of a m/s^2. The acceleration of M_1 is same as of M_2. Let R be normal reaction and μR be the friction force acting on M_1. T be the tension in the string.

Writing the equations of motion for mass M_1

$\Sigma F_x = ma_x$: $\quad M_1 a = T - \mu R \quad$ ...(*i*)

$\Sigma F_y = ma_y$: $\quad 0 = R - M_1 g \quad$ ...(*ii*)

Writing the equation of motion for the mass M_2

$\Sigma F_y = ma_y$: $\quad M_2 a = M_2 g - T$

Or $\quad T = M_2(g - a) \quad$ ...(*iii*)

From (*i*) and (*ii*) $\quad M_1 a = T - \mu (M_1 g) \quad$ ...(*iv*)

Or $\quad T = M_1(a + \mu g)$

Equating (*ii*) and (*iv*)

$$M_1(a + \mu g) = M_2(g - a)$$

$$M_1 a + M_1 \mu g = M_2 g - M_2 a$$

$$a(M_2 + M_1) = M_2 g - M_1 \mu g$$

Or

$$a = g(M_2 - \mu M_1)/(M_1 + M_2)$$

Substituting for a in (*iii*)

$$T = M_2(g - a)$$

$$T = M_2\left(g - \frac{g(M_2 - \mu M_1)}{M_1 + M_2}\right)$$

$$T = \frac{M_2 g}{M_1 + M_2}(M_1 + M_2 - M_2 + \mu M_1)$$

$$T = \frac{M_2 g M_1}{M_1 + M_2}(1 + \mu)$$

$$T = \frac{M_1 M_2 g}{(M_1 + M_2)}(1 + \mu)$$

Substituting, $M_1 = 10$ kg, $M_2 = 5$ kg, $\mu = 0.25$ m

$$a = \frac{g(M_2 - \mu M_1)}{(M_1 + M_2)} = \frac{g[5 - 0.25(10)]}{(10 + 5)}$$

$a = 1.635 \text{ m/s}^2$ *Ans.*

$$T = \frac{10 \times 5 \times 9.81}{(10 + 5)}(1 + 0.25)$$

$T = 40.875$ N *Ans.*

Q.6. A) A spring of stiffness 1000 N/m is stretched by 10 cm from the unreformed position. Find the work of the spring force. Also find the work required to stretch it by another 10 cm. (6)

Solution.

$$k = 1000 \text{ N/m}$$
$$x_1 = 10 \text{ cm} = 0.1 \text{ m}$$
$$x_2 = 20 \text{ cm} = 0.2 \text{ m}$$

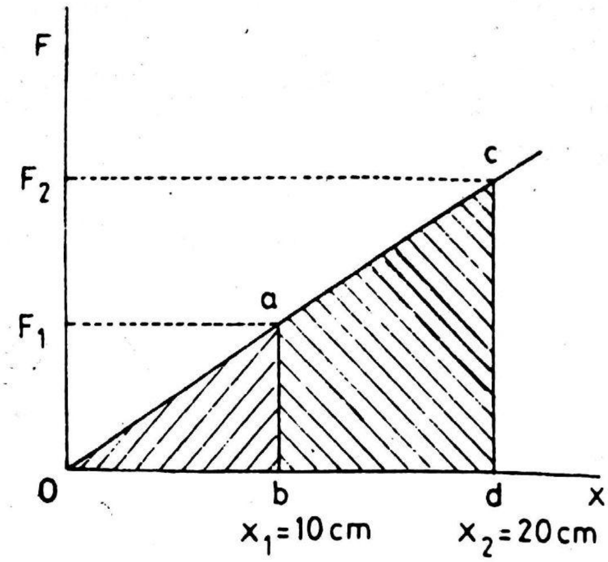


Fig. 16.7

Work required to stretch the spring by 10 cm from the undeformed position

$$U_{0-10} = -\frac{k}{2}(x_1^2 - x_0^2)$$

$$U_{0-10} = -\frac{k}{2}x_1^2$$

$$U_{0-10} = -\frac{1}{2} \times 1000(0.1)^2$$

$$U_{0-10} = -5 \text{ Nm} \quad \textit{Ans.}$$

U_{0-10} is the area of the triangle *oab*.

Work required to stretch from 10 cm to 20 cm

$$U_{10-20} = -\frac{1}{2}k(x_2^2 - x_1^2)$$

$$U_{10-20} = -\frac{1}{2} \times 1000(0.2^2 - 0.1^2)$$

$$U_{10-20} = -15 \text{ Nm} \quad \textit{Ans.}$$

U_{10-20} is the area of the trapezoid *abcd*.

It may be noted here that work required to stretch the spring first 10 cm is 5 Nm and for next 10 cm it is 15 Nm.

B) What do you understand by direct central impact? Also explain the coefficient of restitution. (6)

that may act ...

18.2 DIRECT CENTRAL IMPACT

Consider the two spheres A and B of masses m_a and m_b moving in the same direction and along the same straight line with the known velocities v_a and v_b respectively.

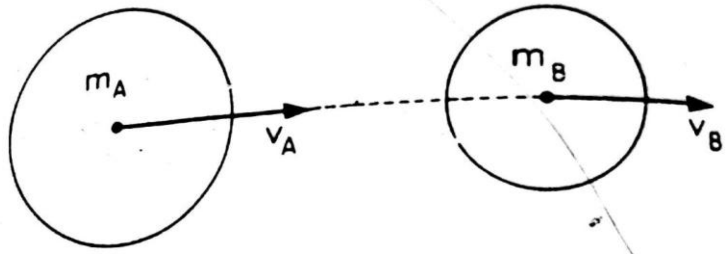


Fig. 18.2

If $v_a > v_b$, the sphere A will strike the sphere B.

Since no external force is acting, the total momentum of the system of spheres A and B is conserved.

$$m_a v_a + m_b v_b = m_a v_a' + m_b v_b' \quad ...(18.1)$$

where, v_a' and v_b' are the velocities after the impact.

As all velocities are directed along the line of impact they can be treated as scalars. For the purpose of fixing the sense of velocity and momentum. They are taken as positive when directed to the right.

Now, it is required to determine the velocities v_a' and v_b' after the impact. A single equation obtained above is not sufficient to determine two unknowns. One more equation or additional information is needed to solve for two unknowns. The other equation can be derived based on the nature of impact and is

$$e = -\frac{(v_b' - v_a')}{v_b - v_a} \quad ...(18.2)$$

where, e is called the *coefficient of restitution* and its value depends upon the nature of impact. In a problem the value e is either given for an impact or is to be determined.

$$e = (-)\frac{\text{velocity of separation}}{\text{velocity of approach}} \quad ...(18.3)$$

For an exact definition of the coefficient of restitution we have to look into the nature of impact which is discussed in the next section.

With these two equations now, we can solve for the unknown velocities v_a' and v_b' if the value of e is known.