

**DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY,
LONERE - RAIGAD - 402 103
Winter Semester Examination - December - 2017**

Branch: M.Tech. (Chemical)

Semester: I

Subject with Subject Code:-MCH102

Marks: 60

Date:- 18/12/17

Time:3 Hrs.

Instructions to the Students

1. Each question carries 12 marks.
2. Attempt any five questions of the following.
3. Illustrate your answers with neat sketches, diagram etc., wherever necessary.
4. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly.
5. Use the given formula sheet wherever necessary.

(Marks)

Q.1	<p>A) The mathematical expressions of the thermal conditions at the boundaries are called the boundary conditions. To describe a heat transfer problem completely, two boundary conditions must be given for each direction of the coordinate system along which heat transfer is significant. Therefore, we need to specify four boundary conditions for two-dimensional problems.</p>	2
	<p>B) Heat transfer to a canned drink can be modeled as two-dimensional since temperature differences (and thus heat transfer) will exist in the radial and axial directions (but there will be symmetry about the center line and no heat transfer in the azimuthal direction. This would be a transient heat transfer process since the temperature at any point within the drink will change with time during heating. Also, we would use the cylindrical coordinate system to solve this problem since a cylinder is best described in cylindrical coordinates. Also, we would place the origin somewhere on the center line, possibly at the center of the bottom surface.</p>	2
	<p>C) Derive an expression for the temperature distribution in a steady state plane wall having uniformly distributed heat sources and one face maintained at a temperature T_1 while the other face is maintained at a temperature T_2. The thickness of the wall may be taken as $2L$.</p> <p style="text-align: center;">Uniformly distributed heat sources</p> $\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \quad T = T_1 \quad \text{at} \quad x = -L$ $T = T_2 \quad \text{at} \quad x = +L$ $T = -\frac{\dot{q}x^2}{2k} + c_1x + c_2$ $T_1 = -\frac{\dot{q}L^2}{2k} - c_1L + c_2$ $T_2 = -\frac{\dot{q}L^2}{2k} + c_1L + c_2$ $T = \frac{\dot{q}}{2k}(L^2 - x^2) + \frac{T_2 - T_1}{2L}x + \frac{T_1 + T_2}{2}$	1 1 1 1
	<p>D) A thick-walled copper cylinder has an inside radius of 1cm and an outside radius of 1.8 cm. The inner and outer surface temperatures are held at 305°C and 295°C, respectively. Assume k varies linearly with temperature. A reported value of k for this material at 150 °C is 371.9 W/m.K</p>	

and b is $-9.25 \times 10^{-5} \text{ K}^{-1}$. Determine the heat loss **per unit length**.

We have to take mean thermal conductivity equation for hollow cylinder $q = -k(T)AdT/dr$

$$\frac{q}{L} = -2\pi k_m \frac{T_2 - T_1}{\ln(r_2/r_1)}$$

where $k_m = k_0[1 + b\theta_m]$. For this problem,

$$\theta_m = \frac{\theta_2 + \theta_1}{2} = \frac{(305 - 150) + (295 - 150)}{2} = 150 \text{ }^\circ\text{C}$$

and from Problem 2.16,

$$k_m = (371.9) \text{ W/m}\cdot\text{K}[1 - 9.25 \times 10^{-5} \text{ K}^{-1}(150) \text{ }^\circ\text{C}] = 366.7 \text{ W/m}\cdot\text{K}$$

Hence,

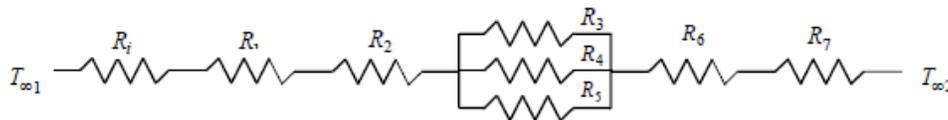
$$\frac{q}{L} = -2\pi \left(366.7 \frac{\text{W}}{\text{m}\cdot\text{K}} \right) \frac{(295 - 305) \text{ K}}{\ln(1.8/1)} = 39.20 \text{ kW/m}$$

Q.2 A)

Assumptions 1 Heat transfer is steady since there is no indication of change with time. 2 Heat transfer through the wall is one-dimensional. 3 Thermal conductivities are constant. 4 Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be $k = 0.72 \text{ W/m}\cdot\text{ }^\circ\text{C}$ for bricks, $k = 0.22 \text{ W/m}\cdot\text{ }^\circ\text{C}$ for plaster layers, and $k = 0.026 \text{ W/m}\cdot\text{ }^\circ\text{C}$ for the rigid foam.

Analysis We consider 1 m deep and 0.33 m high portion of wall which is representative of the entire wall. The thermal resistance network and individual resistances are



$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot \text{ }^\circ\text{C})(0.33 \times 1 \text{ m}^2)} = 0.303 \text{ }^\circ\text{C/W}$$

$$R_1 = R_{foam} = \frac{L}{kA} = \frac{0.02 \text{ m}}{(0.026 \text{ W/m}\cdot\text{ }^\circ\text{C})(0.33 \times 1 \text{ m}^2)} = 2.33 \text{ }^\circ\text{C/W}$$

$$R_2 = R_6 = R_{plaster, side} = \frac{L}{kA} = \frac{0.02 \text{ m}}{(0.22 \text{ W/m}\cdot\text{ }^\circ\text{C})(0.30 \times 1 \text{ m}^2)} = 0.303 \text{ }^\circ\text{C/W}$$

$$R_3 = R_5 = R_{plaster, center} = \frac{L}{h_o A} = \frac{0.18 \text{ m}}{(0.22 \text{ W/m}\cdot\text{ }^\circ\text{C})(0.015 \times 1 \text{ m}^2)} = 54.55 \text{ }^\circ\text{C/W}$$

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{0.18 \text{ m}}{(0.72 \text{ W/m}\cdot\text{ }^\circ\text{C})(0.30 \times 1 \text{ m}^2)} = 0.833 \text{ }^\circ\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(20 \text{ W/m}\cdot\text{ }^\circ\text{C})(0.33 \times 1 \text{ m}^2)} = 0.152 \text{ }^\circ\text{C/W}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{54.55} + \frac{1}{0.833} + \frac{1}{54.55} \longrightarrow R_{mid} = 0.81 \text{ }^\circ\text{C/W}$$

$$R_{total} = R_i + R_1 + 2R_2 + R_{mid} + R_o = 0.303 + 2.33 + 2(0.303) + 0.81 + 0.152 = 4.201 \text{ }^\circ\text{C/W}$$

The steady rate of heat transfer through the wall per 0.33 m^2 is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[(22 - (-4))]^\circ\text{C}}{4.201 \text{ }^\circ\text{C/W}} = 6.19 \text{ W}$$

Then steady rate of heat transfer through the entire wall becomes

$$\dot{Q}_{total} = (6.19 \text{ W}) \frac{(4 \times 6) \text{ m}^2}{0.33 \text{ m}^2} = 450 \text{ W}$$

Or

1

A)

Assumptions 1 Steady operating conditions exist. 2 Heat transfer is one-dimensional since heat transfer from the side surfaces are disregarded 3 Thermal conductivities are constant.

Properties The thermal conductivities are given to be $k = 223 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}$ for copper and $0.15 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}$ for epoxy layers.

Analysis We take the length in the direction of heat transfer to be L and the width of the board to be w . Then heat conduction along this two-layer plate can be expressed as (we treat the two layers of epoxy as a single layer that is twice as thick)

$$\dot{Q} = \dot{Q}_{\text{copper}} + \dot{Q}_{\text{epoxy}} = \left(kA \frac{\Delta T}{L} \right)_{\text{copper}} + \left(kA \frac{\Delta T}{L} \right)_{\text{epoxy}} = [(kt)_{\text{copper}} + (kt)_{\text{epoxy}}] w \frac{\Delta T}{L}$$

Heat conduction along an "equivalent" plate of thickness $t = t_{\text{copper}} + t_{\text{epoxy}}$ and thermal conductivity k_{eff} can be expressed as

$$\dot{Q} = \left(kA \frac{\Delta T}{L} \right)_{\text{board}} = k_{\text{eff}} (t_{\text{copper}} + t_{\text{epoxy}}) w \frac{\Delta T}{L}$$

Setting the two relations above equal to each other and solving for the effective conductivity gives

$$k_{\text{eff}} (t_{\text{copper}} + t_{\text{epoxy}}) = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \longrightarrow k_{\text{eff}} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}}$$

Note that heat conduction is proportional to kt . Substituting, the fraction of heat conducted along the copper layer and the effective thermal conductivity of the plate are determined to be

$$(kt)_{\text{copper}} = (223 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F})(0.03/12 \text{ ft}) = 0.5575 \text{ Btu/h}\cdot^\circ\text{F}$$

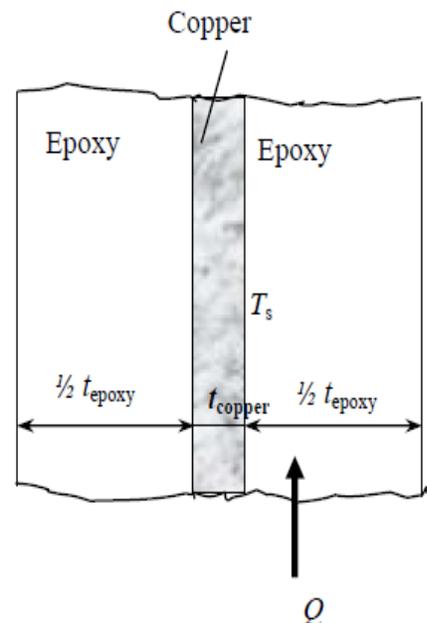
$$(kt)_{\text{epoxy}} = 2(0.15 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F})(0.1/12 \text{ ft}) = 0.0025 \text{ Btu/h}\cdot^\circ\text{F}$$

$$(kt)_{\text{total}} = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} = (0.5575 + 0.0025) = 0.56 \text{ Btu/h}\cdot^\circ\text{F}$$

and

$$k_{\text{eff}} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}} = \frac{0.56 \text{ Btu/h}\cdot^\circ\text{F}}{[(0.03/12) + 2(0.1/12)] \text{ ft}} = 29.2 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}$$

$$f_{\text{copper}} = \frac{(kt)_{\text{copper}}}{(kt)_{\text{total}}} = \frac{0.5575}{0.56} = 0.996 = 99.6\%$$



2

2

1

2

2

2

Q.3 A)

Assumptions 1 The egg is spherical in shape with a radius of $r_0 = 2.75$ cm. 2 Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. 3 The thermal properties of the egg are constant. 4 The heat transfer coefficient is constant and uniform over the entire surface. 4 The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs are given to be $k = 0.6$ W/m.°C and $\alpha = 0.14 \times 10^{-6}$ m²/s.

Analysis The Biot number for this process is

$$Bi = \frac{hr_0}{k} = \frac{(1400 \text{ W/m}^2 \cdot \text{°C})(0.0275 \text{ m})}{(0.6 \text{ W/m} \cdot \text{°C})} = 64.2$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

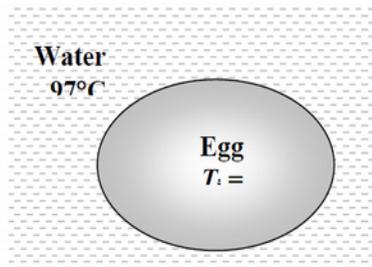
$$\lambda_1 = 3.0877 \text{ and } A_1 = 1.9969$$

Then the Fourier number becomes

$$\theta_{0, sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70 - 97}{8 - 97} = (1.9969)e^{-(3.0877)^2 \tau} \longrightarrow \tau = 0.198 \approx 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the time required for the temperature of the center of the egg to reach 70°C is determined to be

$$t = \frac{\tau r_0^2}{\alpha} = \frac{(0.198)(0.0275 \text{ m})^2}{(0.14 \times 10^{-6} \text{ m}^2/\text{s})} = 1068 \text{ s} = \mathbf{17.8 \text{ min}}$$



B) The Heisler charts of Figures 4-7(Holman J.P) and 4-10 (Holman J.P) may be used for solution of this problem. We first calculate the center temperature of the plate, using Figure 4-7, and then use Figure 4-10 to calculate the temperature at the specified x position. From the conditions of the problem we have,

$\theta_i = T_i - T_\infty = 200 - 70 = 130 \quad \alpha = 8.4 \times 10^{-5} \text{ m}^2/\text{s} \quad [3.26 \text{ ft}^2/\text{h}]$
 $2L = 5.0 \text{ cm} \quad L = 2.5 \text{ cm} \quad \tau = 1 \text{ min} = 60 \text{ s}$
 $k = 215 \text{ W/m} \cdot \text{°C} \quad [124 \text{ Btu/h} \cdot \text{ft} \cdot \text{°F}]$
 $h = 525 \text{ W/m}^2 \cdot \text{°C} \quad [92.5 \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{°F}]$
 $x = 2.5 - 1.25 = 1.25 \text{ cm}$

Then

$$\frac{\alpha \tau}{L^2} = \frac{(8.4 \times 10^{-5})(60)}{(0.025)^2} = 8.064 \quad \frac{k}{hL} = \frac{215}{(525)(0.025)} = 16.38$$

$$\frac{x}{L} = \frac{1.25}{2.5} = 0.5$$

From Graph of Midplane temperature for an infinite plate of thickness 2L: (θ) full scale.

$$\frac{\theta_0}{\theta_i} = 0.61$$

$$\theta_0 = T_0 - T_\infty = (0.61)(130) = 79.3$$

Temperature as a function of center temperature in an infinite plate of thickness 2L

$$x/L = 0.5, \quad \frac{\theta}{\theta_0} = 0.98$$

$$\theta = T - T_\infty = (0.98)(79.3) = 77.7$$

$$T = 77.7 + 70 = 147.7^\circ\text{C}$$

1

1

2

1

1

Q.4**A)**
Nitrogen gas, vertical wall so,**4**

$$\frac{\bar{h}L}{k} = \overline{\text{Nu}} = C(\text{Gr}_L \text{Pr})^a$$

For the gas, $\beta = 1/T = 1/300 = 3.33 \times 10^{-3} \text{ }^\circ\text{K}^{-1}$, giving a Grashof number of

$$\begin{aligned} \text{Gr}_L &= \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \\ &= \frac{(9.8 \text{ m/s}^2)(3.33 \times 10^{-3} \text{ }^\circ\text{K}^{-1})[(50 - 4) \text{ K}](1.8 \text{ m})^3}{(15.63 \times 10^{-6})^2 \text{ m}^4/\text{s}^2} = 3.58 \times 10^{10} \end{aligned}$$

and

$$\text{Gr}_L \text{Pr} = (3.58 \times 10^{10})(0.713) = 2.55 \times 10^{10}$$

The flow is turbulent; (8.28) with the appropriate constants from Table 8-3 gives

$$\frac{\bar{h}L}{k} = (0.13)(\text{Gr}_L \text{Pr})^{1/3}$$

where

$$\bar{h} = \frac{0.0262 \text{ W/m}\cdot\text{K}}{1.8 \text{ m}} (0.13)(2.55 \times 10^{10})^{1/3} = 5.570 \text{ W/m}^2\cdot\text{K}$$

and the heat loss is given by

$$q = \bar{h}A(T_s - T_\infty) = \left(5.570 \frac{\text{W}}{\text{m}^2\cdot\text{K}}\right)(1.8 \text{ m})(2.45 \text{ m})[(50 - 4) \text{ }^\circ\text{C}] = 1130 \text{ W}$$

B)

$$L_c = (D_o - D_i)/2 = (0.3 - 0.2)/2 = 0.05 \text{ m}$$

The Rayleigh number is

$$\begin{aligned} \text{Ra}_L &= \frac{g\beta(T_i - T_o)L^3}{\nu^2} \text{Pr} \\ &= \frac{(9.81 \text{ m/s}^2)[1/(300 \text{ K})](320 - 280 \text{ K})(0.05 \text{ m})^3}{(1.58 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.729) = 4.776 \times 10^5 \end{aligned}$$

The effective thermal conductivity is

$$\begin{aligned} F_{\text{sph}} &= \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5} \\ &= \frac{0.05 \text{ m}}{[(0.2 \text{ m})(0.3 \text{ m})]^4 [(0.2 \text{ m})^{-7/5} + (0.3 \text{ m})^{-7/5}]^5} = 0.005229 \end{aligned}$$

$$\begin{aligned} k_{\text{eff}} &= 0.74k \left(\frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{sph}} \text{Ra}_L)^{1/4} \\ &= 0.74(0.02566 \text{ W/m} \cdot ^\circ\text{C}) \left(\frac{0.729}{0.861 + 0.729} \right) (0.005229 \times 4.776 \times 10^5)^{1/4} \\ &= 0.1104 \text{ W/m} \cdot ^\circ\text{C} \end{aligned}$$

Then the rate of heat transfer between the spheres becomes

$$\begin{aligned} \dot{Q} &= k_{\text{eff}} \pi \left(\frac{D_i D_o}{L_c} \right) (T_i - T_o) \\ &= (0.1104 \text{ W/m} \cdot ^\circ\text{C}) \pi \left(\frac{(0.2 \text{ m})(0.3 \text{ m})}{0.05 \text{ m}} \right) (320 - 280) \text{ K} = \mathbf{16.7 \text{ W}} \end{aligned}$$

Therefore, heat will be lost from the inner sphere to the outer one at a rate of 16.7 W.

4

C) The Prandtl number $\text{Pr} = \nu/\alpha$ has been introduced in the above expressions along with a new dimensionless group called the *Grashof number* Gr_x :

$$\text{Gr}_x = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}$$

The Grashof number may be interpreted physically as a dimensionless group representing the ratio of the buoyancy forces to the viscous forces in the free-convection flow system. It has a role similar to that played by the Reynolds number in forced-convection systems and is the primary variable used as a criterion for transition from laminar to turbulent boundarylayer flow. For air in free convection on a vertical flat plate, the critical Grashof number has been observed by Eckert and Soehngen [1] to be approximately 4×10^8 . Values ranging between 108 and 109 may be observed for different fluids and environment “turbulence levels.”

for free convection from vertical and inclined surfaces to water under constant-heat-flux conditions, the results are presented in terms of a modified Grashof number, Gr_x^* :

$$\text{Gr}_x^* = \text{Gr}_x \text{Nu}_x = \frac{g\beta q_w x^4}{k\nu^2}$$

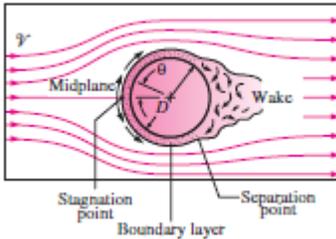
where $qw = q/A$ is the heat flux per unit area and is assumed constant over the entire plate surface area.

4

Q.5

A) For Force Convection, Explain flow across cylinders and spheres. What is the effect of surface roughness on the friction drag coefficient in laminar and turbulent flows?

Theory J.P.Holman



roughening the surface can be used to great advantage in reducing drag, but it can also backfire on us if we are not careful—specifically, if we do not operate in the right range of Reynolds number. With this consideration, golf balls are intentionally roughened to induce *turbulence* at a lower Reynolds number to take advantage of the sharp *drop* in the drag coefficient at the onset of turbulence in the boundary layer (the typical velocity range of golf balls is 15 to 150 m/s, and the Reynolds number is less than 4×10^5). The critical Reynolds number of dimpled golf balls is about 4×10^4 . The occurrence of turbulent flow at this Reynolds number reduces the drag coefficient of a golf ball by half,

5

B)

the fluid properties for glycerin, given in Table B-3 (Engl.), are:

$$\rho = (16.01846)(78.91) = 1264 \text{ kg/m}^3 \quad k = (1.729577)(0.165) = 0.2854 \text{ W/m}\cdot\text{K}$$

$$\nu = (0.0929)(0.0127) = 0.00118 \text{ m}^2/\text{s} \quad \text{Pr} = 12500$$

The critical length is

$$x_c = \frac{\nu}{V_\infty} \text{Re}_c = \frac{0.00118 \text{ m}^2/\text{s}}{3 \text{ m/s}} (500000) = 196.7 \text{ m}$$

Therefore, the flow is laminar throughout, and the average heat transfer coefficient is given by (6.28) as

$$\frac{\bar{h}L}{k} = (0.664)\text{Re}_L^{1/2} \text{Pr}^{1/3}$$

The Reynolds number at the end of the plate is

$$\text{Re}_L = \frac{V_\infty L}{\nu} = \frac{(3 \text{ m/s})(1 \text{ m})}{0.00118 \text{ m}^2/\text{s}} = 2542$$

Hence

$$\bar{h} = \frac{0.2854 \text{ W/m}\cdot\text{K}}{1 \text{ m}} (2542)^{1/2} (12500)^{1/3} = 333.9 \text{ W/m}^2\cdot\text{K}$$

and the heat transfer is given by

$$\frac{q}{W} = \bar{h}L(T_s - T_\infty) = \left(333.9 \frac{\text{W}}{\text{m}^2\cdot\text{K}}\right) (1 \text{ m})[(30 - 10) \text{ K}] = 6.679 \text{ kW/m}$$

(b) For ammonia at 20°C, properties from Table B-3 (Engl.) are

$$\rho = (16.01846)(38.19) = 611.75 \text{ kg/m}^3 \quad k = (1.729577)(0.301) = 0.521 \text{ W/m}\cdot\text{K}$$

$$\nu = (0.0929)(0.386 \times 10^{-5}) = 3.59 \times 10^{-7} \text{ m}^2/\text{s} \quad \text{Pr} = 2.02$$

and the critical length is

$$x_c = \frac{\nu}{V_\infty} \text{Re}_c = \frac{3.59 \times 10^{-7} \text{ m}^2/\text{s}}{3 \text{ m/s}} (500000) = 0.0598 \text{ m}$$

The Reynolds number at the end of the plate is

$$\text{Re}_L = \frac{V_\infty L}{\nu} = \frac{(3 \text{ m/s})(1 \text{ m})}{3.59 \times 10^{-7} \text{ m}^2/\text{s}} = 8.36 \times 10^6$$

and the average heat transfer coefficient is given by (7.28).

$$\frac{\bar{h}L}{k} = \text{Pr}^{1/3} (0.036 \text{Re}_L^{0.8} - 836)$$

$$\bar{h} = \frac{0.521 \text{ W/m}\cdot\text{K}}{1 \text{ m}} (2.02)^{1/3} [(0.036)(8.36 \times 10^6)^{0.8} - 836]$$

$$= (0.6586)[12418 - 836] = 7.628 \text{ kW/m}^2\cdot\text{K}$$

In this case the reduction due to the laminar portion at the leading edge is minor, being only 7.2% of the total.

The heat transfer is

$$\frac{q}{W} = \bar{h}L(T_s - T_\infty) = \left(7.628 \frac{\text{kW}}{\text{m}^2\cdot\text{K}}\right) (1 \text{ m})[(30 - 10) \text{ K}] = 153 \text{ kW/m}$$

Q.6

A) Some surfaces encountered in numerous practical heat transfer applications are modeled as being adiabatic as the back sides of these surfaces are well insulated and net heat transfer through these surfaces is zero. When the convection effects on the front (heat transfer) side of such a surface is negligible and steady-state conditions are reached, the surface must lose as much radiation energy as it receives. Such a surface is called reradiating surface. In radiation analysis,

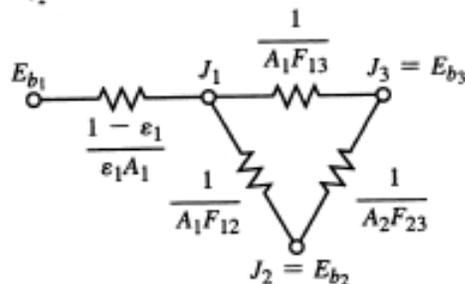
12

the surface resistance of a reradiating surface is taken to be zero since there is no heat transfer through it.

B)

$$A_1 = A_2 = (0.9)(0.6) = 0.54 \text{ m}^2 \quad \epsilon_1 = 0.6 \quad E_{b_1} = 23,220 \text{ W/m}^2$$

$$E_{b_2} = 401 \text{ W/m}^2 \quad A_3 \rightarrow \infty \quad F_{12} = 0.25 \quad F_{13} = 0.75 = F_{23}$$



$$\frac{1 - \epsilon_1}{\epsilon_1 A_1} = 1.235 \quad \frac{1}{A_1 F_{13}} = \frac{1}{A_2 F_{23}} = 2.469 \quad \frac{1}{A_1 F_{12}} = 7.407$$

$$q = \frac{23,220 - 401}{1.235 + \frac{1}{\frac{1}{2.469} + \frac{1}{7.407}}} = 7108 \text{ W} = \frac{23,220 - J_1}{1.235}$$

$$J_1 = 14,441 \text{ W/m}^2 \quad \frac{14,441 - J_2}{7.407} = \frac{J_2 - 401}{2.469}$$

$$J_2 = 3911 = E_{b_2} = \sigma T_2^4 \quad T_2 = 512.5 \text{ K} = 239.5^\circ\text{C}$$

C)

SOLUTION The temperature of the filament of an incandescent lightbulb is given. The fraction of visible radiation emitted by the filament and the wavelength at which the emission peaks are to be determined.

Assumptions The filament behaves as a blackbody.

Analysis The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.4 \mu\text{m}$ to $\lambda_2 = 0.76 \mu\text{m}$. Noting that $T = 2500 \text{ K}$, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 11-2 to be

$$\lambda_1 T = (0.40 \mu\text{m})(2500 \text{ K}) = 1000 \mu\text{m} \cdot \text{K} \quad \longrightarrow \quad f_{\lambda_1} = 0.000321$$

$$\lambda_2 T = (0.76 \mu\text{m})(2500 \text{ K}) = 1900 \mu\text{m} \cdot \text{K} \quad \longrightarrow \quad f_{\lambda_2} = 0.053035$$

That is, 0.03 percent of the radiation is emitted at wavelengths less than $0.4 \mu\text{m}$ and 5.3 percent at wavelengths less than $0.76 \mu\text{m}$. Then the fraction of radiation emitted between these two wavelengths is (Fig. 11-15)

$$f_{\lambda_1 - \lambda_2} = f_{\lambda_2} - f_{\lambda_1} = 0.053035 - 0.000321 = 0.0527135$$

Therefore, only about 5 percent of the radiation emitted by the filament of the lightbulb falls in the visible range. The remaining 95 percent of the radiation appears in the infrared region in the form of radiant heat or "invisible light," as it used to be called. This is certainly not a very efficient way of converting electrical energy to light and explains why fluorescent tubes are a wiser choice for lighting.

The wavelength at which the emission of radiation from the filament peaks is easily determined from Wien's displacement law to be

$$(\lambda T)_{\text{max power}} = 2897.8 \mu\text{m} \cdot \text{K} \quad \longrightarrow \quad \lambda_{\text{max power}} = \frac{2897.8 \mu\text{m} \cdot \text{K}}{2500 \text{ K}} = 1.16 \mu\text{m}$$