DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE – RAIGAD -402 103

Mid SemesterExamination – October - 2017

Branch:M.TechVLSI **Sem.:- I**

Subject with Subject Code:-Graph Theory and Discrete Optimization (MTVLC101)

Marks: 20

Date:-09-10-2017 **Time:- 1 Hr.**

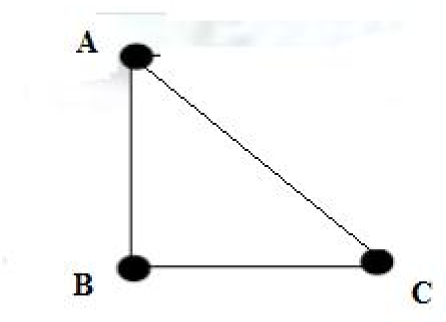
MODEL ANSWER SOLUTION

Instructions:-Assume necessary data wherever required. **(Marks)**

Q.No.1 Attempt any one of the following (08)

a.) What are Hamiltonian circuit, Hamiltonian trail and Hamiltonian graph? Prove that if G fails Chavatal's condition, then G has at least (n – 2) edges. Conclude from this that the maximum number of edges in a simple non-Hamiltoniann-vertex graph is $\binom{n-1}{2} + 1$.

Ans: Hamiltonian trail of a graph is a one which visits every vertex only once and it can start from any vertex and end to any vertex.
Hamiltonian circuit is a closed path that visits every vertex only once, start and end vertex is same.
A connected graph G is said to be a Hamiltonian graph, if there exists a cycle which contains all the vertices of G.



Example:
For the above graph the Hamiltonian trail is A → B. → C → A .
As the trail contains all the vertices it is Hamiltonian graph. The graph start and end on the same vertex so it is Hamiltonian circuit.

Suppose G fails Chavatal's condition. Then there exists some i < n/2 such that $di \geq i$and$dn - i < n - i$. Let u be a vertex with degree di, and let v be a vertex with degree dn-i. Thus, in G, we have

$$d_{\bar{G}}(u) + d_{\bar{G}}(v) = (n-1) - \mathrm{dG(u)} + (n-1) + \mathrm{dG(v)}$$

$$= 2n - 2 - d_i - d_{n-i}$$
$$> 2n - 2 - i - (n - i) = n - 2$$

So

$$d_{\bar{G}}(u) + d_{\bar{G}}(v) \geq n - 1$$

Since u and v have degree sum at least n - 1 in $\bar{G}$, and since a simple graph has at most one edge joining them (counted twice in the degree sum), there must be at least n - 2 distinct edges in G incident to {u,v}. So $|E(\bar{G})| \geq n - 2$.

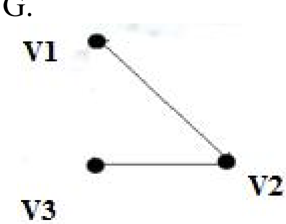
Therefore, the number of edges in a simple non-Hamiltonian graph G on n vertices is

$$|E(G)| = \binom{n}{2} - |E(\bar{G})| \leq \frac{n(n-1)}{2} - (n-2)$$

$$= \frac{n^2-3n+4}{2} = \frac{(n-1)(n-2)}{2} + 1 = \binom{n-1}{2} + 1$$

So the maximum number of edges is $\binom{n-1}{2} + 1$.

b.) What is a spanning tree give example? Find the number of spanning trees for the below incomplete graph G.



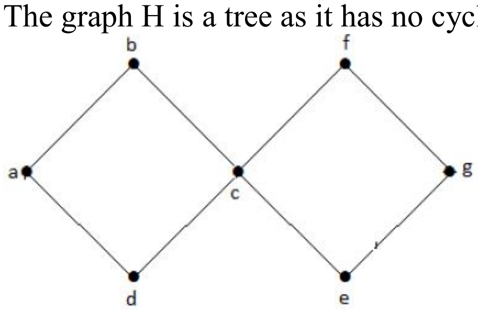
G

Ans:

A spanning tree of an undirected graph is the sub-graph that includes all the vertices of G.

Example: In the below example, G is the connected graph and H is the sub-graph of G.

The graph H is a tree as it has no cycles. Hence it is a spanning tree.



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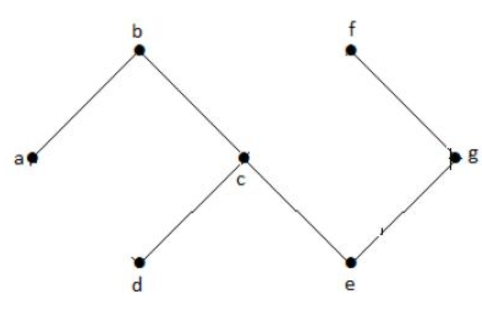
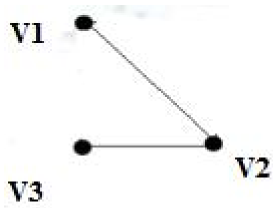


H



Here to find the spanning tree.

1. Construct the adjacency matrix.

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

2. Diagonal 0's should be replaced with degree of node.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

3. Non-diagonal 1's should be replaced with "-1".
4. Non-diagonal 0's will remain 0's.

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

5. No of spanning trees= co-factor of any element.

$$a_{11} = (-1)^{1+1}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

Therefore, No of spanning tree =1

Q.No. 2 Attempt any three of the following: (12)

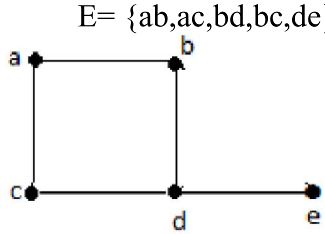
a.)Define.(Give an example)

1.Graph

Ans: A graph is a pair of sets (V, E), where V is the set of vertices and E is the set of edges, connecting the pair of vertices. In the following graph

V= {a,b,c,d,e}

E= {ab,ac,bd,bc,de}

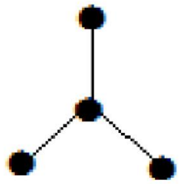


2.Tree

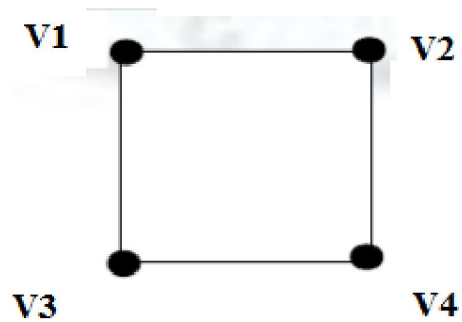
Ans: A connected acyclic graph is called a tree. In other words, a connected graph with no cycles is called a tree.

The edges of a tree are known as **branches**. Elements of trees are called their **nodes**. The nodes without child nodes are called **leaf nodes**.

Example: The graph shown here is a tree because it has no cycles and it is connected. It has four vertices and three edges, i.e., for 'n' vertices 'n-1' edges.



b.) What is an Adjacency matrix and Incidence matrix of graph? Find the adjacency matrix of G shown in figure below



G

Ans:
An adjacency matrix is a way to represent a finite graph G on n vertices by an nxn matrix a, whose entries are either 0 or 1.
Consider a graph G having vertex set V (G) = {V1, V2……Vn}. Then the adjacency matrix A(G) = $[a_{ij}]$ of G is a n x n matrix defined as

$$a_{ij}=\begin{cases}1 & ifviisadjacenttovj \\ 0 & otherwise\end{cases}$$

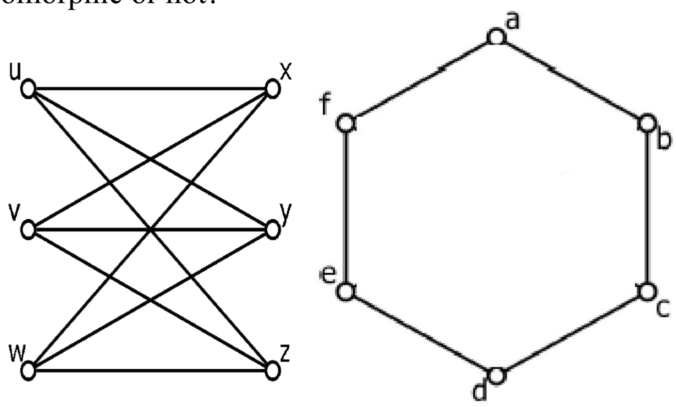
The incidence matrix of a graph is a matrix representation of the graph, in which each row and each column represents an edge and a vertex, respectively.
Consider a graph G having vertex set V (G) = {v1,v2…vn} and E(G) = {e1,e2…..en}. Then the incident matrix B = $[b_{ij}]$ of G is a n x m matrix defined as

$$b_{ij}=\begin{cases}1 & ifviandejareincident \\ 0 & otherwise\end{cases}$$

The adjacency matrix of G is

$$A(G) = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

c.) What is an Isomorphic graph? Prove whether the graphs G1 and G2 are isomorphic or not?



G1 **G2**

Ans:
Two graphs G1 and G2 are said to be isomorphic if:
1. Their numbers of components (vertices and edges) are same.
2. Their edge connectivity is retained.

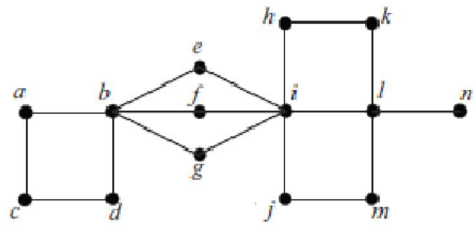
If G1 ≡ G2 then:
1. |V(G1)| = |V(G2)|
2. |E(G1)| = |E(G2)|
3. Degree sequences of G1 and G2 are same.

For the above two graphs G1 and G2
1. Number of vertices are same i.e 6
2. Number of edges for G1 are 9 and for G2 are 6 which are not same.

Therefore, graphs G1 and G2 are non-isomorphic graphs as all the condition for isomorphic graphs are not satisfied.

d.) Define Eccentricity, radius and diameter of graph. Prove that For any connected graph G, rad(G) ≤ diam(G) ≤ 2 rad(G).
Ans:
For a given vertex v of a connected graph, the *eccentricity* of v, denoted ecc(v), is defined to be the greatest distance from v to any other vertex. That is,
ecc(v) = $\max\{d(u, v)\}$
x□V (G)

In Figure below, ecc(a) = 5 since the farthest vertices from a (namely k, m, n) are at a distance of 5 from a. Of the vertices in this graph, vertices c, k,mand n have the greatest eccentricity (6), and vertices e, f and g have the smallest eccentricity (3). These values and types of vertices are given special names.
In a connected graphG, the radius of G, denoted rad(G), is the value of the smallest eccentricity.
The diameter of G, denoted diam(G), is the value of the greatest eccentricity.
In Figure below, the radius is 3, the diameter is 6.



Proof:
By definition, rad(G) ≤ diam(G), so we just need to prove the secondinequality. Let u and v be vertices in G such that d(u, v) = diam(G). Further, let c be a vertex in the center of G. Then,

diam(G) = d(u, v) ≤ d(u, c) + d(c, v) ≤ 2 ecc(c) = 2 rad(G)