Subject: Signal Theory
Solution

Q.1A random variable X has the following probability distribution (08)

x	-2	-1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	3K

Find (i) the value of K (ii) Evaluate $P(X < 2)$ and $P(-2 < X < 2)$ (iii) find the cdf of X

Answer:

(i) the value of K
Since P(x) =1, 6K+0.6=1
K=1/15

(ii) Evaluate $P(X < 2)$ and $P(-2 < X < 2)$

x	-2	-1	0	1	2	3
P(x)	0.1	1/15	0.2	2/15	0.3	3/15

$P(X < 2)$ = P(X= -2, -1, 0 or 1)
= P(X= -2) +P(X= -1) +P(X= 0) +P(X= 1)
= 1/2

$P(-2 < X < 2)$ = P(X= -1, 0 or 1)
=P(X= -1) +P(X= 0) +P(X= 1)
= 2/5

(iii) find the cdf of X
F(x)=0, when x< -2
=1/10, when -2≤ x< -1
=1/6, when -1≤ x< 0
=11/30, when 0≤ x< 1
=1/2, when 1≤ x< 2
=4/5, when 2≤ x< 3
=1, when 3≤ x

-OR-

Q.1 For the bivariate probability distribution of (X,Y) given below, find $P(X \leq 1)$, $P(Y \leq 3)$, $P(X \leq 1, Y \leq 3)$, *and* $P(X \leq 1/Y \leq 3)$. (08)

Y / X	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

Answer:

$P(X \leq 1) = P(X=0) + P(X=1)$

$= \sum_{j=1}^{6} P(X = 0, Y = j) + \sum_{j=1}^{6} P(X = 1, Y = j)$

= (0+0+1/32+2/32+2/32+3/32) + (1/16+1/16+1/8+1/8+1/8+1/8)

= 7/8

$P(Y \leq 3) = P(Y=1) + P(Y=2) + P(Y=3)$

$= \sum_{i=0}^{2} P(X = i, Y = 1) + \sum_{i=0}^{2} P(X = i, Y = 2) + \sum_{i=0}^{2} P(X = i, Y = 3)$

= 23/64

$P(X \leq 1, Y \leq 3) = \sum_{j=1}^{3} P(X = 0, Y = j) + \sum_{j=1}^{3} P(X = 1, Y = j)$

=9/32

$P(X \leq 1/Y \leq 3) = \frac{P(X \leq 1, Y \leq 3)}{P(Y \leq 3)} = \frac{9/32}{23/64}$

= 18/23

Q.2 Attempt any three

a) Prove that, $E\{(X-a)^2\} = E\{(X-\mu)^2\} + (\mu - a)^2$

Answer:

Let us consider the LHS $E\{(X-\mu)^2\}+(\mu-a)^2$

$E\{X^2-2\mu X+\mu^2)\}+(\mu^2+a^2-2a\mu)$

$E\{X^2)-\mu^2+\mu^2+a^2-2a\mu$

$E\{X^2)+a^2-2a\mu$

Which can be written as

$E\{X^2+a^2-2aX\}$ which is nothing but RHS.

$E\{(X-a)^2\}=E\{(X-\mu)^2\}+(\mu-a)^2$

b) State the axioms of probability. State and prove the theorem of total probability

Answer:

Axioms of probability

(i) $0 \leq P(A) \leq 1$

(ii) $P(S) = 1$

(iii) $P(A \cup B) = P(A) + P(B)$

(iv) If $A_1, A_2, \ldots\ldots.A_n$ are a set of mutually exclusive events then

$P(A_1 \cup A_2 \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots.+ P(A_n)$

Theorem of total probability

If $B_1, B_2, \ldots\ldots., B_n$ be a set of exhaustive and mutually exclusive events then

$$\mathbf{P(A) = \sum_{i=1}^{n} P(Bi)P(A/Bi)}$$

c) State the properties of Probability distribution and density function.

Answer:

Properties of probability distribution functions

(i) $F_x(x)$ **is non-decreasing function of x.**

(ii) $F_x(-\infty) = 0$ **and** $F_x(\infty) = 1$

(iii) $\frac{d}{dx}F_X(x) = f_X(x)$ **at all points, where** $F_X(x)$ **is differentiable.**

(iv) $If\ X$ **is discrete random variable taking values** $x_1, x_2, x_3, \ldots.$ **Where** $x_1 < x_2 < x_3 \ldots, x_{i-1} < x_i < \ldots.$ **then** $P(X=x_i)=F_X(x_i)-F_X(x_{i-1})$

Properties of probability density function

forcontinuous random variable

(i) $f(x) \geq 0$ for all $x \in R_x$

(ii) $\int_{Rx} f(x)\, dx = 1$

for discrete random variable

(i) $P_i \geq 0$ for all i

(ii) $\sum_i$ Pi =1

d) Continuous random Variable has a pdf$f(x) = 3x^2, 0 \leq x \leq 1$. Find a and b such that

(i) $P(X \leq a) = P(X > a)$ and (ii) $P(X > b) = 0.05$

Answer:

(i) $P(X \leq a) = P(X > a)$

$\int_0^a 3x^2 dx = \int_a^1 3x^2 dx$

$a^3 = 1 - a^3$

$a^3 = 1/2$

a=0.7937

(ii) $P(X > b) = 0.05$

$\int_b^1 3x^2 dx = 0.05$

b= 0.9830