DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE – RAIGAD -402 103
Mid Semester Examination – October - 2017

Branch: M.Tech Electrical Drives and Control
Sem.:- I
Subject:- Electrical Machine Modeling and Analysis
Subject Code:-MTEDC102 **Marks:20**
Date:- **Time:- 1 Hr.**

SOLUTION

Q1A:solution

3.4.1. Separately excited d.c. generator

Q1 - 1

The transfer functions for separately excited d.c. generators are obtained first for its no-load operation and then for its on-load operation.

(*a*) **No-load operation.** The equations specifying the no-load operation of a separately excited d.c. generator, from Eq. [3.1 (*a*)], are

$$v_f = (r_f + L_f p)\, i_f$$

and

$$e_a = M_d\, \omega_r\, i_f = K_g\, i_f$$

$$\therefore \quad e_a = \frac{M_d\, \omega_r\, v_f}{r_f(1+\tau_f p)} = \frac{K_g\, v_f}{r_f(1+\tau_f p)}.$$

The transfer function relating the no-load armature voltage e_a to the field voltage v_f, is given by

$$\frac{e_a}{v_f} = \frac{K_g}{r_f}\,\frac{1}{1+\tau_f p}$$

The above transfer function in *s*-domain is

$$\frac{E_a(s)}{V_f(s)} = \frac{K_g}{r_f}\cdot\frac{1}{1+s\tau_f} \qquad \text{...(3.41)}$$

The block diagram representation of Eq. (3.41) is shown in Fig. 3.12 (*a*).

(*b*) **On-load operation.** The voltage equations specifying the on-load operation of the separately excited d.c. generators, from Eq. [3.1(*a*)], are

$$v_f = (r_f + L_f p)\, i_f$$

$$v_t = M_d\, \omega_r\, i_f - (r_a + L_a p)\, i_a.$$

For a load impedance $(R_L + L_L p)$ across the armature terminals, we get

$$v_t = (R_L + L_L p)\, i_a$$

$$\therefore \quad v_t = (R_L + L_L p)\, i_a = K_g\, i_f - (r_a + L_a p)\, i_a$$

or

$$i_a\,[(r_a + R_L) + (L_a + L_L)\, p] = K_g\, i_f$$

or $$i_a = \frac{K_g\, i_f}{(r_a + R_L)(1 + \tau)}$$

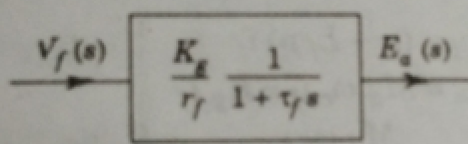
where $$\tau = \frac{L_a + L_L}{r_a + R_L}$$

From above, the armature current in s-domain is

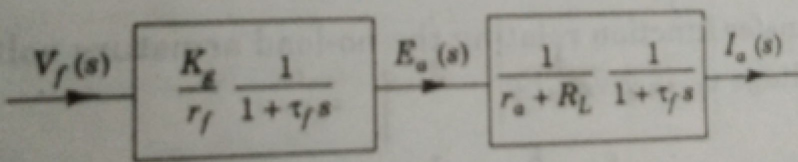
$$I_a(s) = \frac{K_g \cdot V_f(s)}{r_f(1 + s\tau_f)(r_a + R_L)(1 + s\tau)}$$

Therefore, the transfer function relating the armature current $I_a(s)$ to the field winding voltage $V_f(s)$ is

$$\frac{I_a(s)}{V_f(s)} = \frac{K_g}{r_f} \cdot \frac{1}{1 + s\tau_f} \cdot \frac{1}{(r_a + R_L)(1 + s\tau)} \quad \text{...(3.42)}$$



(a)



(b)

Q1 B-Solution

2.3. Transformation from Three Phases to Two Phases (a, b, c to α, β, 0)

Q1 - 2

A symmetrical 2-pole, 3-phase winding on the rotor is represented by three coils *A, B, C* each of *N* effective turns and mutually displaced by 120°, see Fig. 2.3 (*a*). Maximum values of m.m.fs. F_a, F_b and F_c are shown along their respective phase-axes. The combined effect of these three m.m.fs, results in a constant magnitude m.m.f., which rotates at a constant angular velocity depending on the poles and frequency.

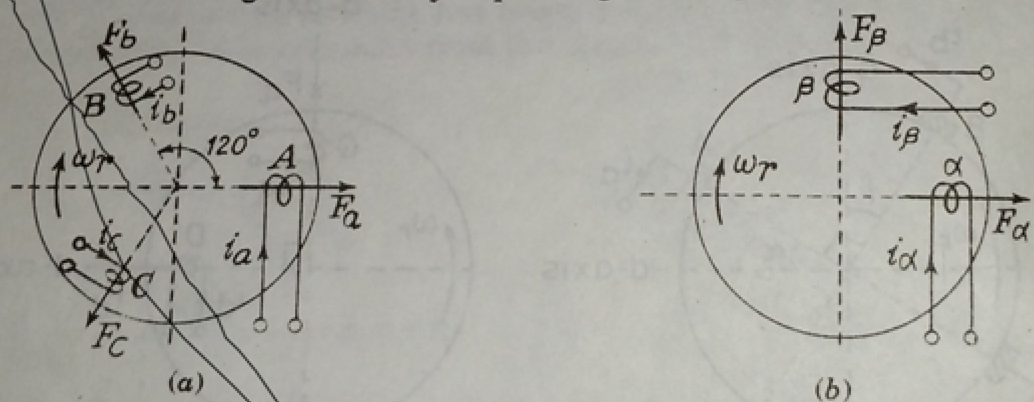


Fig. 2.3. (*a*) Balanced 3-phase windings. (*b*) 2-phase windings on the rotor.

If the three currents are

$$i_a = I_m \cos \omega t$$

$$i_b = I_m \cos\left(\omega t - \frac{2\pi}{3}\right)$$

and

$$i_c = I_m \cos\left(\omega t - \frac{4\pi}{3}\right)$$

then, these will produce a m.m.f. of constant magnitude $\frac{3I_m N}{2}$ rotating with respect to the three-phase winding at the time frequency. The space angle between the windings must comply with the time-phase angle between the currents.

In Fig. 2.3 (*b*), a balanced two-phase winding is represented by two orthogonal coils α, β on the rotor. For convenience in transformation, the axes of phase '*A*' and 'α' are taken to be coincident.

If two-phase currents

$$i_\alpha = I_m \cos \omega t$$

and

$$i_\beta = I_m \cos\left(\omega t - \frac{\pi}{2}\right) = I_m \sin \omega t$$

flow in the two-phase windings, the result will be a m.m.f. of constant magnitude $I_m N$, revolving with respect to the two-phase windings at the time frequency of the phase currents.

The m.m.f.s of three-phase and two-phase systems can be rendered equal in magnitude by making any one of the following changes :

(*i*) by changing magnitude of the two-phase currents,

(*ii*) by changing number of turns of the two phase windings,

(*iii*) by changing both the magnitude of currents and number of turns.

These will now be discussed one after the other.

(*i*) If the effective number of turns per phase in case of two-phase winding is N (*i.e.* the winding factors are same for both three- and two-phase windings) then for equal m.m.fs the magnitude of the current in the two phases, must be 3/2 times the magnitude of the three-phase currents. This can be proved by resolving the instantaneous 3-phase m.m.fs along the α-β axis. Reference to Fig. 2.3, gives

$$i_\alpha N = N[i_a \cos 0° + i_b \cos 120° + i_c \cos 240°]$$

or

$$i_\alpha = \left[i_a - \frac{1}{2}(i_b + i_c)\right]$$

Similarly $\quad i_\beta N = N\,[i_a \sin 0° + i_b \sin 120° + i_c \sin 240°]$

or

$$i_\beta = \left[0 + \frac{\sqrt{3}}{2} i_b - \frac{\sqrt{3}}{2} i_c\right\} \qquad \text{...(2.3)}$$

For a balanced system

$$i_a + i_b + i_c = 0$$

$$\therefore \qquad i_\alpha = \left[i_a - \frac{1}{2}(i_b + i_c)\right]$$

or

$$i_\alpha = i_a + \frac{1}{2} i_a \quad \text{or} \quad i_\alpha = \frac{3}{2} i_a \qquad \text{...(2.4)}$$

Thus the magnitude of the two-phase currents is 3/2 times the magnitude of the three-phase currents.

per phase being the same in both the windings, the magnitude of phase e.m.fs of the two and three-phase windings would be equal. The power per phase of the two-phase system ($V \cdot 3/2 \cdot I$) is thus 3/2 times the power per phase ($V \cdot I$) of the three phase system. Note that the total power of the two-phase system ($= 2 \cdot V \cdot 3/2 \cdot I$) and three-phase system ($= 3\ VI$) is the same. Thus the invariance of power has been attained. The only disadvantage is that the transformation of current and voltage will differ because of the presence of factor 3/2 in the current transformation. As factor 3/2 appears in current transformation and not in voltage transformation, the per phase parameters of the two-phase and three-phase induction machines will not be the same.

(*ii*) If the effective number of per-phase turns of the two-phase winding is made 3/2 times that of the three phase winding, then for equal m.m.fs the magnitude of the currents in the two phase and three phase systems must be equal *i.e.* $i_\alpha = i_a$. This can be proved as is done in the previous case.

With these conditions, the per phase voltage of two phase machine will be 3/2 times ($3/2\ V$) the per phase voltage (V) of the three phase system.

The power per phase in two-phase system $= \dfrac{3}{2} VI$

Total power in the two-phase system $= 3\ VI$

Also total power in the three-phase system $= 3\ VI$

Here again the invariance of power has been obtained, but, as earlier, the transformation of current and voltage will differ because of the factor 3/2 in the voltage transformation. As such, per phase parameters of the 2-phase induction machine will be different from that of the 3-phase induction machine.

(*iii*) Here both the magnitude of currents and number of turns of the two-phase system are changed to obtain identical transformation for voltage and current.

Let the number of per-phase turns in the two-phase winding be made $\sqrt{\dfrac{3}{2}}$ times the per-phase turns of the three-phase winding. Then for equal m.m.fs, $i_\alpha = \sqrt{\dfrac{3}{2}}\, i_a$. This can be proved by resolving the 3-phase m.m.fs along the α-axis as shown below :

$$i_\alpha \cdot \sqrt{\frac{3}{2}}\, N = N\left[i_a - \frac{1}{2} i_b - \frac{1}{2} i_c\right]$$

or
$$i_\alpha = \sqrt{\frac{2}{3}}\left[i_a - \frac{1}{2}(i_b + i_c)\right]$$
$$= \sqrt{\frac{2}{3}}\left[i_a + \frac{1}{2} i_a\right]$$
$$= \sqrt{\frac{3}{2}}\, i_a \qquad \ldots(2.5)$$

The voltage per phase of the two-phase winding is $\sqrt{\frac{3}{2}}$ times that of the three-phase winding. Thus the phase voltage and current of the two-phase system are $\sqrt{\frac{3}{2}}$ times those of the three-phase system. This fact results in identical transformations for both the voltage and current.

$$\therefore \quad i_\alpha = \sqrt{\frac{2}{3}}\left(i_a - \frac{1}{2} i_b - \frac{1}{2} i_c\right) \qquad \ldots(2.6)$$
and
$$i_\beta = \sqrt{\frac{2}{3}}\left(0 + \frac{\sqrt{3}}{2} i_b - \frac{\sqrt{3}}{2} i_c\right)$$

Since the transformations for voltage and current are identical, impedance per phase of the two- and three-phase systems is the same. This is illustrated by an example given below :

Q2A Solution

3.5. D.C. Series Motor

Q2 – 9

The arrangement of this machine is shown in Fig. 3.17 (*a*) and its primitive in Fig. 3.17 (*b*).

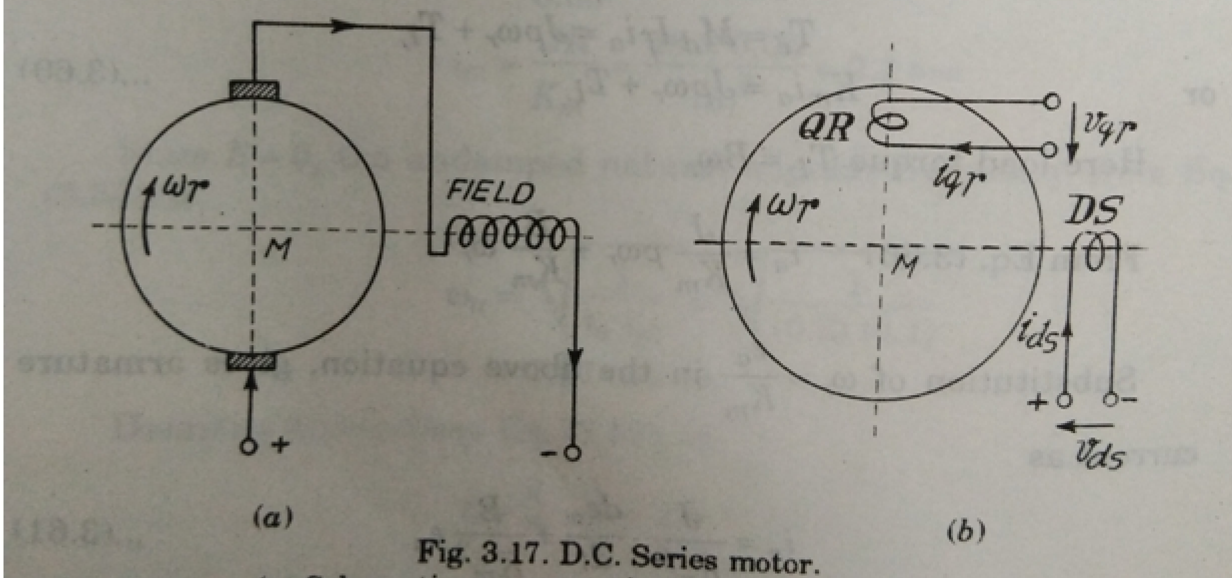


Fig. 3.17. D.C. Series motor.
(*a*) Schematic representation and (*b*) Its primitive.

The voltage equations are

$$\begin{array}{c} \\ \\ \end{array}\begin{bmatrix} v_{ds} \\ v_{qr} \end{bmatrix} = \begin{array}{c|cc} & ds & qr \\ \hline ds & r_{ds}+L_{ds}\,p & \\ qr & M_d\,\omega_r & r_{qr}+L_{qr}\,p \end{array} \begin{bmatrix} i_{ds} \\ i_{qr} \end{bmatrix} \quad ...(3.1)$$

But in case of series commutator machines, the field and armature are connected in series as shown in Fig. 3.18.

$\therefore \quad i_{ds} = i_{qr} = i_f$

$\qquad = i_a = i$ (say)

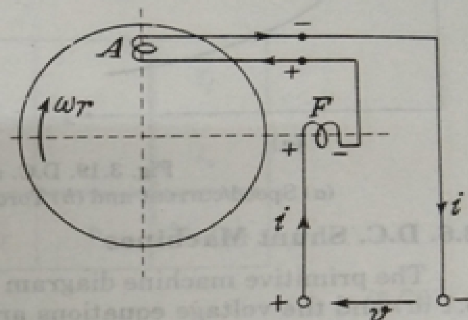


Fig. 3.18. Interconnection of field and armature coils for series motor.

With this change, voltage equations become

$$v_f = (r_f + L_f p)\, i$$

$$v_a = M_d\,\omega_r\, i + (r_a + L_a\, p)\, i$$

But $v = v_a + v_f$

$$\therefore \ v = [(r_a + r_f) + (L_a + L_f)\,p + M_d\,\omega_r]\, i \quad ...(3.63)$$

or $$v = (R + Lp + M_d\,\omega_r)\, i \quad ...[3.63\,(a)]$$

$$\text{Torque matrix } [G] = \begin{bmatrix} & \\ M_d & \end{bmatrix}$$

$\therefore$ Electro-magnetic torque

$$T_e = M_d\, i^2 = Jp\omega_r + D\omega_r + T_L \quad ...(3.64)$$

Equations (3.63) and (3.64) are equally valid for d.c. and a.c. single phase series machines.

Steady state analysis. For d.c. machines, p is replaced by zero for steady state conditions.

$$\therefore \quad V = (R + M_d\,\omega_r)\, I$$

or $$\omega_r = \frac{V - IR}{M_d\, I} \cong \frac{V}{M_d\, I} \quad ...(3.65)$$

Steady state torque

$$T_e = M_d\, I^2 = \frac{V^2}{M_d\,\omega_r^{\,2}} \quad ...(3.66)$$

Eq. (3.65) gives speed-current curve of Fig. 3.19 (*a*) and Eq. (3.66) gives torque-current curve of Fig. 3.19 (*b*).

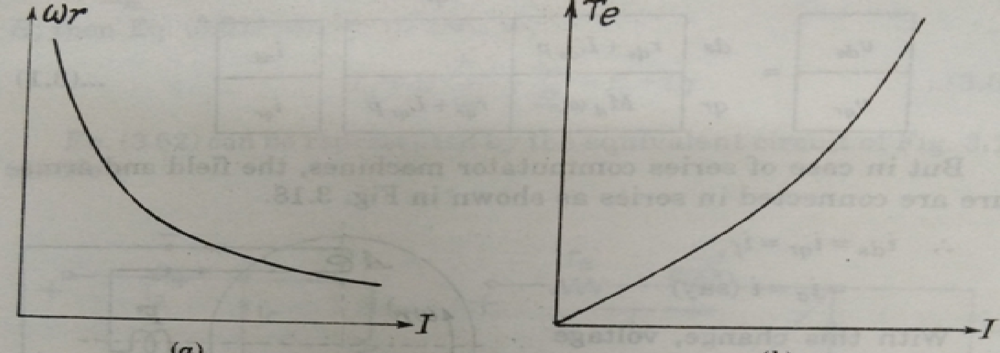


Fig. 3.19. D.C. series motor.
(*a*) Speed/current and (*b*) Torque/current characteristics.

Q2B Solutionl

- ***Magnetically coupled*** **circuit means that two loops, with or without contacts between them, affect each other through the magnetic field generated by one of them.**

Self Inductance

i_2 +

−

$$v_1 = -M \frac{di_2}{dt}$$

Q2-(3) Solution

It has been stated earlier that the primitive machine can be shown equivalent to any of the rotating electrical machines. The rotor winding of the primitive machine is fitted with commutator and brushes and the rotor magnetic field is space and time invariant. The electrical machines, most closely resembling the primitive machine are d.c. machines, since these are also constructed with commutator plus brushes and the field produced in them is space and time invariant. Thus the d.c. machines are equivalent to the primitive machine with appropriate number of coils on each fixed axis and no transformation is necessary.

A wide variety of rotating machines are polyphase a.c. machines constructed in a different manner than the primitive machine. In such cases, the primitive machine can also be employed in the analysis, provided the rotating polyphase winding on the rotor and the stationary polyphase winding on the stator can be represented by the *d-q* axes coils of the primitive machine. The process of replacing one set of variables by another related set of variables is called *winding transformation* or merely, transformation.

The term "linear transformation" means that the transformation from old to new set of variables or *vice-versa*, is governed by linear equations*. The idea of using linear transformations, for simplifying the solution, is not new to the reader. He must have used logarithms for doing the multiplication or division of two or more numbers. For example, for multiplying two numbers, we take the $\log_{10}$ of both the numbers, add them and then take the inverse by the use of logarithm tables to get the required multiplication. This is done primarily for simplifying the process of multiplication; logarithm is thus an example of linear transformation. Similarly, the Laplace transform is

Q2D:solution

also a linear transformation. It transforms the time-domain (old variables) equation to s-domain (new variables) equation and after manipulations, we again get the required time-domain solution. The equations relating the symmetrical components of currents with 3-phase currents are equivalent to a linear transformation. The process of referring secondary quantities to primary or primary quantities to secondary in a transformer is also equivalent to a linear transformation. Note that the transformation from old to new set of variables is carried out merely for making the calculations simpler and less laborious.

The equations expressing old variables in terms of new variables or *vice versa* are called *transformation equations.* These equations, when written in matrix, have the following general form :

[New variables] = [Transformation matrix] [Old variables]

or [Old variables] = [Transformation matrix] [New variables]

The *transformation matrix* may, therefore, be defined as a matrix containing the coefficients relating the old and new variables.

Linear transformations in electrical machines are usually carried out for the purpose of obtaining new equations which are fewer in number or are more easily solved. For example, a 3-phase machine requires three voltage equations whereas its generalized model requires only two voltage equations which can be solved more easily as compared to three voltage equations. Further, the circuit equations for a 3-phase machine are more complicated because of the magnetic coupling amongst the three-phase windings, but this is not the case in the generalized (or two-axis) model, in which m.m.f. acting along one axis has no mutual coupling with the m.m.f. acting along the other axis.

A transformation can be arbitrary, selected purely on mathematical grounds to get a suitable matrix, which results in simplifying the process of obtaining the solution of a problem. However, most of the transformations used here have physical meaning, though it is not necessary for a transformation to correspond to any physical equivalent.

Q2

Example 3.7. *A 10 kW, 230 V, 1500 rpm d.c. motor has the following constants :*

Q2-4.

$$r_a = 1.00\ \Omega, \qquad L_a = 0.10\ H,$$
$$K_m = M_d I_f = 4.00\ Nm/armature\ amp.$$
$$J = 1.00\ kg\text{-}m^2.$$

The load coupled with the motor has its inertia equal to 1.00 kg-m^2. If load torque varies linearly with speed, then calculate ω_n, ξ and investigate its dynamic behaviour. Neglect rotational losses.

Solution.

Here $$\omega_r = \frac{2\pi \times 1500}{60} = 157.1 \text{ rad/sec}$$

Rated load torque,

$$T_e = T_L = \frac{P}{\omega_r} = \frac{10{,}000}{157.1} = 63.65 \text{ Nm.}$$

Since torque-speed curve is a straight line through origin, the slope of the torque-speed curve of the load is

$$B = \frac{T_L}{\omega_r} = \frac{63.65}{157.1} = 0.405\ ;\ \frac{B}{J} = 0.2025$$

$$\tau_a = \frac{L_a}{r_a} = 0.1 \text{ sec.}$$

$$\tau_m = \frac{J r_a}{K_m^2} = \frac{2 \times 1}{(4.0)^2} = 0.125$$

From Eq. (3.51),

$$\omega_n = \sqrt{\frac{1}{\tau_a}\left(\frac{B}{J} + \frac{1}{\tau_m}\right)}$$

$$= \sqrt{\frac{1}{0.1}\left(0.2025 + \frac{1}{0.125}\right)} = 9.057 \text{ rad/sec.}$$

From Eq. (3.52), $$\alpha = \frac{1}{2}\left(\frac{B}{J} + \frac{1}{\tau_a}\right)$$

$$= \frac{1}{2}(0.2025 + 10) = 5.101.$$

From Eq. (3.53), $$\xi = \frac{a}{\omega_n} = \frac{5.101}{9.057} = 0.5632.$$

Since the desirable range of ξ for practical purposes is from 0.3 to 0.7, the dynamic behaviour of this motor system is a favourable one. The maximum speed deviation, from normal speed, during sudden load applications is well within limits for this value of damping ratio.