Subject: Signal Processing Algorithms and Applications

Solution

Que.1 An FIR filter of length- 3 is defined by a symmetric impulse response; that is, h(n). Let the input to this filter be a sum of two cosine sequences of angular frequencies 0.25π radians/samples and $0.75\,\pi$ radians/samples, respectively. Determine the impulse response coefficients so that the filter passes only the high-frequency component of the input.

Answer: **h(n)=$\{\boldsymbol{h_0}, \boldsymbol{h_1}, \boldsymbol{h_2}\}$**

Since h(n) is symmetric.

$$\boldsymbol{h_2 = h_0}$$

h(n)=$\{\boldsymbol{h_0}, \boldsymbol{h_1}, \boldsymbol{h_0}\}$

$$H\,(e^{jw}) = \boldsymbol{h_0} + \boldsymbol{h_1}\,e^{-jw} + \boldsymbol{h_0}\,e^{-2jw}$$

$$= e^{-jw}[2\boldsymbol{h_0}\boldsymbol{cos}(\boldsymbol{w}) + \boldsymbol{h_1}]$$

$$\left|H\,(e^{jw})\right| = 2\boldsymbol{h_0}\boldsymbol{cos}(\boldsymbol{w}) + \boldsymbol{h_1}$$

$$\left|H\,(e^{j\frac{\pi}{4}})\right| = 2\boldsymbol{h_0}\boldsymbol{cos}\left(\frac{\boldsymbol{\pi}}{\boldsymbol{4}}\right) + \boldsymbol{h_1} = \boldsymbol{0} \text{.........} \textbf{eq(1)}$$

$$\left|H\,(e^{j\frac{3\pi}{4}})\right| = 2\boldsymbol{h_0}\boldsymbol{cos}\left(\frac{\boldsymbol{3\pi}}{\boldsymbol{4}}\right) + \boldsymbol{h_1} = \boldsymbol{1} \text{ ....} \textbf{eq(2)}$$

By solving Eq (1) and (2)

$$\boldsymbol{h_0} = -\frac{\boldsymbol{1}}{\boldsymbol{2\sqrt{2}}}, \qquad \boldsymbol{h_1} = \frac{\boldsymbol{1}}{\boldsymbol{2}}, \quad \boldsymbol{h_2} = -\frac{\boldsymbol{1}}{\boldsymbol{2\sqrt{2}}}$$

-OR-

Q.1 Find thelinear convolution of following sequences using DFT.

x(n)={1,2,2} and h(n)={ 2,1}

<u>Answer:</u>For linear convolution, if the length of given sequences doesn't match then first match the length of sequences by padding zeros using

N=N1+N2-1   where N1= length of first sequence

N2= length of second sequence

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -1-2j \\ 1 \\ -1+2j \end{bmatrix}$$

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2-j \\ 1 \\ 2+j \end{bmatrix}$$

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{bmatrix} = \begin{bmatrix} X(0)H(0) \\ X(1)H(1) \\ X(2)H(2) \\ X(3)H(3) \end{bmatrix} = \begin{bmatrix} 15 \\ -4-3j \\ 1 \\ -4+3j \end{bmatrix}$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 2 \end{bmatrix}$$

Q.2 Attempt any three (12)

a) Obtain a lattice realization of the function

$$H(z) = 1 + 0.2 z^{-1} + 0.3 z^{-2}$$

<u>Answer:</u> Since the order of function is 2we need to find $H_1(z)$ for which reflection coefficient, $K_2 = h_2$

$$h'_i = \frac{h_i - K_M\, h_{M-i}}{K_M^2}$$

$$h'_1 = \frac{h_1 - h_2\, h_1}{h_2^2}$$

$$H_1(z) = 1 + h'_1 Z^{-1}$$

b) Derive the formula for Cutoff frequency of first order IIR low pass filter

Answer:

$$H_{LP}(Z) = \frac{1-\propto}{z} \cdot \frac{1+z^{-1}}{1-\propto z^{-1}}$$

$$\text{Cut of frequency}\,\omega_c = cos^{-1}\left(\frac{2\propto}{1+\propto^2}\right)$$

$$\text{Where } \propto = \frac{1 \pm \sin(\omega c)}{\cos(\omega c)}$$

c)Let $X(e^{jw})$ denote the Fourier transform of the signal $x[n]$. Perform the following calculations without explicitly evaluating $X(e^{jw})$:

i) Find $X(e^{j\pi})$

ii) Evaluate $\int_{-\pi}^{\pi} \left|X(e^{jw})\right|^2 dw$

For given sequence, $x[n] = \{1, 2, 4, \underset{\uparrow}{2}, 1, -2, \ 2\}$ sequence is starting from n=-3

Answer:

$X(e^{j\pi})$ = (2+2-2) – (1+4+1+2)= 2 – 8 = - 6

Using Parsevals theorem we can write

$$\int_{-\pi}^{\pi} \left|X(e^{jw})\right|^2 dw = 2\pi \sum_{n=-3}^{3} x^2(n) = 68\,\pi$$

d) Show that following transfer function is an Allpass filter

$$H(z) = \frac{b + a\,z^{-1}}{a + b\,z^{-1}}$$

, where a and b are real constants.

Answer:

$$H(z) = \frac{b + a z^{-1}}{a + b z^{-1}}$$

$$H(z)\ H(z^{-1}) = \frac{b+az^{-1}}{a+bz^{-1}}.\ \frac{b+az}{a+bz}$$

$$H(z)\ H(z^{-1})|_{z=e^{jw}} = \frac{b+ae^{-jw}}{a+be^{-jw}}\frac{b+ae^{jw}}{a+be^{jw}}$$

$$= \frac{b^2+a\ b\ e^{jw}+a\ b\ e^{-jw}+a^2}{a^2+a\ b\ e^{jw}+a\ b\ e^{-jw}+b^2}$$

$$=1$$

e) Using overlap and add method perform the linear convolution of following sequences

x(n) ={1, 2, 3, 4} and h(n) ={5, 6}

Answer:

Let x(n) is divided into length two sequence

$x_0(n)$ ={1, 2}

$x_1(n)$ ={3, 4}

$y(n) = \sum_{m=0}^{1} y_m\ (n-2m) = y_0\ (n) + y_1\ (n\text{-}2)$

y_0 (n) = 5, 16, 12

y_1 (n) = 15, 38, 24

y(n) = 5,16, 27, 38, 24