DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE – RAIGAD -402 103
Mid Semester Examination – October - 2017

Branch:M.Tech (CSE) **Sem.:- I**

Subject with Subject Code:- Answer of Modelling and Dynamic System (MTCS101) **Marks: 20**

Q.No.1 Attempt any one of the following (08)

a.)

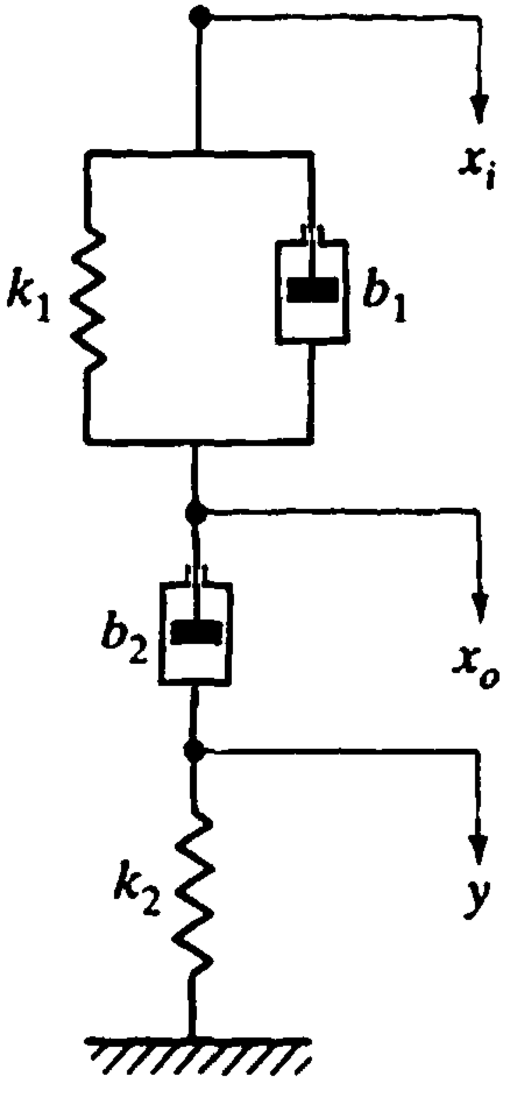


Figure 1.

Solution The equations of motion for the mechanical system are

$$b_1(\dot{x}_i - \dot{x}_o) + k_1(x_i - x_o) = b_2(\dot{x}_o - \dot{y})$$
$$b_2(\dot{x}_o - \dot{y}) = k_2 y$$

Taking the $\mathscr{L}_-$ transform of these two equations, with the initial conditions $x_i(0-) = 0$, $x_o(0-) = 0$ and $y(0-) = 0$, we get

$$b_1[sX_i(s) - sX_o(s)] + k_1[X_i(s) - X_o(s)] = b_2[sX_o(s) - sY(s)]$$
$$b_2[sX_o(s) - sY(s)] = k_2Y(s)$$

If we eliminate $Y(s)$ from the last two equations, the transfer function $X_o(s)/X_i(s)$ becomes

$$\frac{X_o(s)}{X_i(s)} = \frac{\left(\frac{b_1}{k_1}s + 1\right)\left(\frac{b_2}{k_2}s + 1\right)}{\left(\frac{b_1}{k_1}s + 1\right)\left(\frac{b_2}{k_2}s + 1\right) + \frac{b_2}{k_1}s}$$

Substitution of the given numerical values into the transfer function yields

$$\frac{X_o(s)}{X_i(s)} = \frac{(s + 1)(2s + 1)}{(s + 1)(2s + 1) + 4s} = \frac{s^2 + 1.5s + 0.5}{s^2 + 3.5s + 0.5}$$

For an input $x_i(t) = X_i \cdot 1(t)$, the response $x_o(t)$ can be obtained as follows: Since

$$X_o(s) = \frac{s^2 + 1.5s + 0.5}{s^2 + 3.5s + 0.5}\frac{X_i}{s}$$
$$= \left(\frac{0.6247}{s + 3.3508} - \frac{0.6247}{s + 0.1492} + \frac{1}{s}\right)X_i$$

we find that

$$x_o(t) = (0.6247e^{-3.3508t} - 0.6247e^{-0.1492t} + 1)X_i$$

Notice that $x_o(0+) = X_i$.

b.)

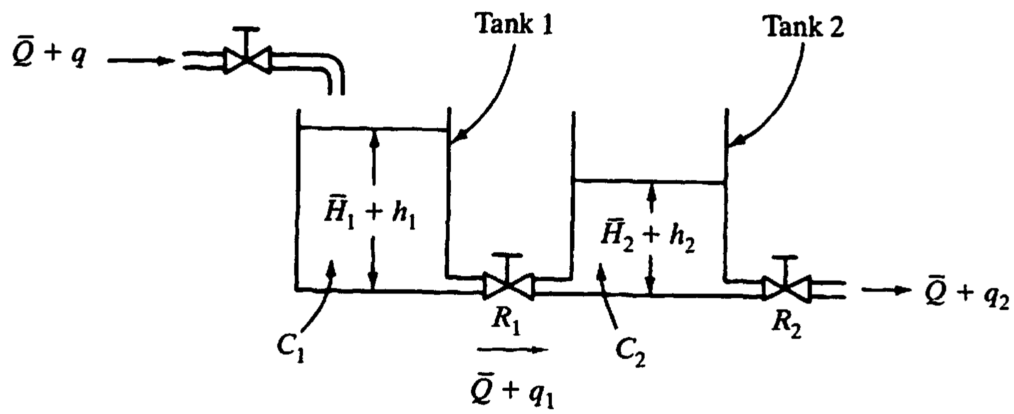


Figure 2.

Solution For tank 1, we have

$$q_1 = \frac{h_1 - h_2}{R_1}$$

$$C_1 \frac{dh_1}{dt} = q - q_1$$

Hence,

$$C_1 \frac{dh_1}{dt} + \frac{h_1}{R_1} = q + \frac{h_2}{R_1} \tag{7–41}$$

For tank 2, we get

$$q_2 = \frac{h_2}{R_2}$$

$$C_2 \frac{dh_2}{dt} = q_1 - q_2$$

Therefore,

$$C_2 \frac{dh_2}{dt} + \frac{h_2}{R_1} + \frac{h_2}{R_2} = \frac{h_1}{R_1} \tag{7–42}$$

Taking Laplace transforms of both sides of Equations (7–41) and (7–42), under the initial conditions $h_1(0) = 0$ and $h_2(0) = 0$, we obtain

$$\left(C_1 s + \frac{1}{R_1}\right) H_1(s) = Q(s) + \frac{1}{R_1} H_2(s) \tag{7–43}$$

$$\left(C_2 s + \frac{1}{R_1} + \frac{1}{R_2}\right) H_2(s) = \frac{1}{R_1} H_1(s) \tag{7–44}$$

From Equation (7–43), we have

$$(R_1 C_1 s + 1) H_1(s) = R_1 Q(s) + H_2(s)$$

or

$$H_1(s) = \frac{R_1 Q(s) + H_2(s)}{R_1 C_1 s + 1}$$

Substituting this last equation into Equation (7–44) yields

$$\left(C_2 s + \frac{1}{R_1} + \frac{1}{R_2}\right) H_2(s) = \frac{1}{R_1} \frac{R_1 Q(s) + H_2(s)}{R_1 C_1 s + 1}$$

Since $H_2(s) = R_2 Q_2(s)$, we get

$$\left(C_2 s + \frac{1}{R_1} + \frac{1}{R_2}\right) R_2 Q_2(s) = \frac{Q(s)}{R_1 C_1 s + 1} + \frac{R_2}{R_1} \frac{Q_2(s)}{R_1 C_1 s + 1}$$

which can be simplified to

$$[(C_2 R_2 s + 1)(R_1 C_1 s + 1) + R_2 C_1 s] Q_2(s) = Q(s)$$

Thus, the transfer function $Q_2(s)/Q(s)$ can be given by

$$\frac{Q_2(s)}{Q(s)} = \frac{1}{R_1 C_1 R_2 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_2 C_1) s + 1}$$

Q.No. 2 Attempt any three of the following: (12)

a.) Give the comparison of Resistance and Capacitance in Electrical, Liquid level and Thermal systems.

Solution:

Resistance and capacitance of liquid level system.

The resistance R of a linear resistor is given by $R = e_R/i$

where e_R is the voltage across the resistor and i is the current through the resistor.

The unit of resistance is the ohm, Resistors do not store electric energy in any form, but instead dissipate it as heat. Note that real resistors may not be linear and may also exhibit some capacitance and inductance effects.

Two conductors separated by a nonconducting medium form a capacitor, so two metallic plates separated by a very thin dielectric material form a capacitor. The capacitance C is a measure of the quantity of charge that can be stored for a given voltage across the plates. The capacitance C of a capacitor can thus be given by

$C = q/e_c$

where q is the quantity of charge stored and e_c is the voltage across the capacitor. The unit of capacitance is the farad (F), where farad = ampere-second/ volt = coulomb/volt. Although a pure capacitor stores energy and can release all of it, real capacitors exhibit various losses. These energy losses are indicated by a power factor, which is the ratio of the energy lost per cycle of ac voltage to the energy stored per cycle. Thus, a small-valued power factor is desirable.

Resistance and capacitance of liquid level system.

Consider the flow through a short pipe with a valve connecting two tanks, as shown in Figure (a). The resistance R for liquid flow in such a pipe or restriction is defined as the change in the level difference (the difference of the liquid levels of the two tanks) necessary to cause a unit change in flow rate; that is, R= change in level difference, m/ change in flow rate, m3/s. Since the relationship between the flow rate and the level difference differs for laminar flow and turbulent flow, we shall consider both cases in what follows. Consider the liquid-level system shown in Figure (b). In this system, the liquid spouts through the load valve in the side of the tank. If the flow through the valve is laminar, the relationship between the steady-state flow rate and the steady state head at the level of the restriction is given by Q= K/H, where

Q = steady-state liquid flow rate, m3/s
K/ = constant, m2/s
H = steady-state head, m
For laminar flow, the resistance R_J is
R/= dH/ dQ =1/ K_J = H/ Q
The laminar-flow resistance is constant and is analogous to the electrical resistance.

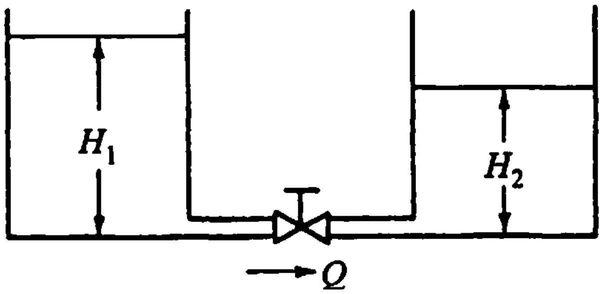


Figure (a)

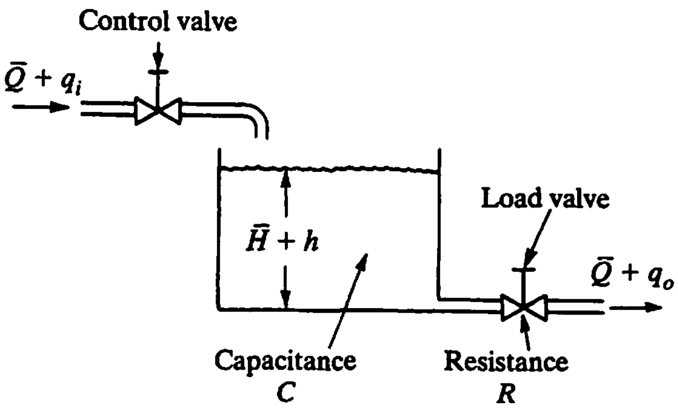


Figure (b)

Thermal resistance and thermal capacitance. The thermal resistance R for heat transfer between two substances may be defined as follows:
R= change in temperature difference °C / change in heat flow rate kcalls
Thus, the thermal resistance for conduction or convection heat transfer is given by R = d(118) = ~ dq K
Since the thermal conductivity and convection coefficients are almost constant, the thermal resistance for either conduction or convection is constant. The thermal capacitance C is defined by
C== change in heat stored kcal / change in temperature °C
Accordingly, the thermal capacitance is the product of the specific heat and the mass of the material. Therefore, thermal capacitance can also be written as

C = mc
where
m = mass of substance considered, kg
c = specific heat of substance, kca/kg °C

b.)

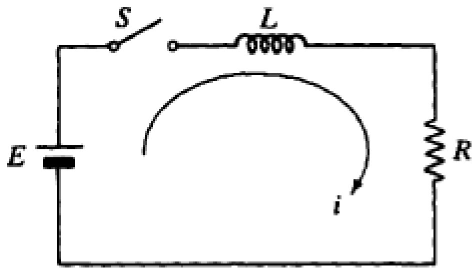


Figure 3.

By arbitrarily choosing the direction of the current around the loop as shown in the figure, we obtain

$$E - L\frac{di}{dt} - Ri = 0$$

or

$$L\frac{di}{dt} + Ri = E \qquad (6\text{–}3)$$

This is a mathematical model for the given circuit. Note that at the instant switch S is closed the current $i(0)$ is zero, because the current in the inductor cannot change from zero to a finite value instantaneously. Thus, $i(0) = 0$.

Let us solve Equation (6–3) for the current $i(t)$. Taking the Laplace transforms of both sides, we obtain

$$L[sI(s) - i(0)] + RI(s) = \frac{E}{s}$$

Noting that $i(0) = 0$, we have

$$(Ls + R)I(s) = \frac{E}{s}$$

or

$$I(s) = \frac{E}{s(Ls + R)} = \frac{E}{R}\left[\frac{1}{s} - \frac{1}{s + (R/L)}\right]$$

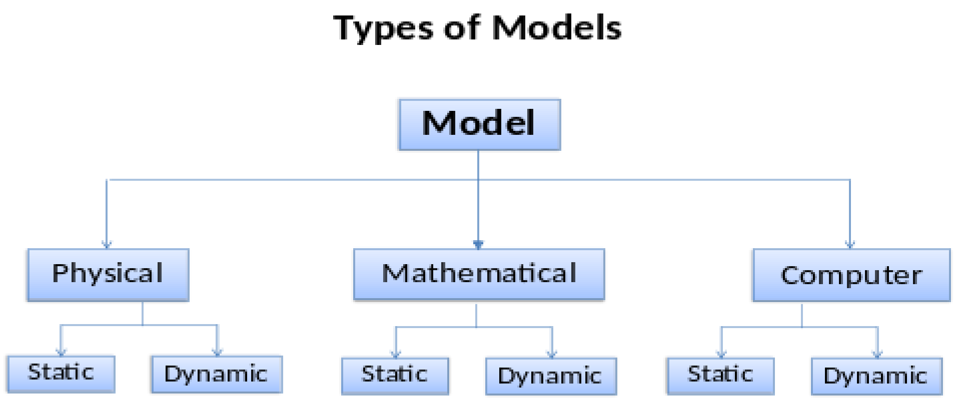
The inverse Laplace transform of this last equation gives

$$i(t) = \frac{E}{R}[1 - e^{-(R/L)t}]$$

c.)

A *model* is a simplified representation or abstraction of reality. Reality is generally too complex to copy exactly. Much of the complexity is actually irrelevant in problem solving.

Mathematical model is a set of mathematical equations (e.g., differential eqs.) that describes the input-output behavior of a system.



Classification of Mathematical Models:

- Linear vs. Non-linear
- Deterministic vs. Probabilistic (Stochastic)
- Static vs. Dynamic
- Discrete vs. Continuous
- White box, black box and gray box

d.)

SIX steps involved in dynamic system approach are as follows:

1. Define the system and its components.
2. Formulate the mathematical model and list the necessary assumptions.
3. Write the differential equations describing the model.
4. Solve the equations for the desired output variables.
5. Examine the solutions and the assumptions.
6. If necessary, reanalyze or redesign the system.