

# MODEL ANSWER OF MME1102

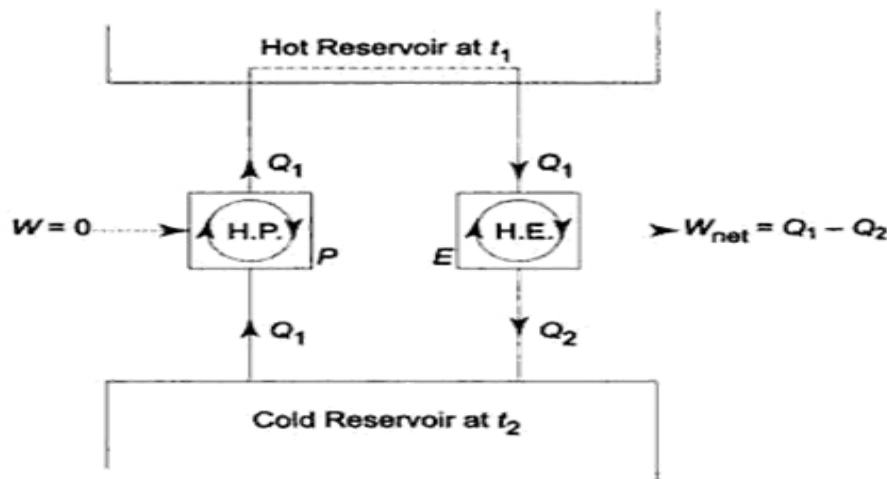
## ENGINEERING THERMODYNAMICS

Q1.a)

The *Kelvin-Planck statement* of the second law states: *It is impossible for a heat engine to produce net work in a complete cycle if it exchanges heat only with bodies at a single fixed temperature.*

Clausius' statement of the second law gives: *It is impossible to construct a device which, operating in a cycle, will produce no effect other than the transfer of heat from a cooler to a hotter body.*

The equivalence of the two statements will be proved if it can be shown that the violation of one statement implies the violation of the second, and vice versa. (a) Let us first consider a cyclic heat pump  $P$  which transfers heat from a low temperature reservoir ( $t_2$ ) to a high temperature reservoir ( $t_1$ ) with no other effect, i.e., with no expenditure of work, violating Clausius statement



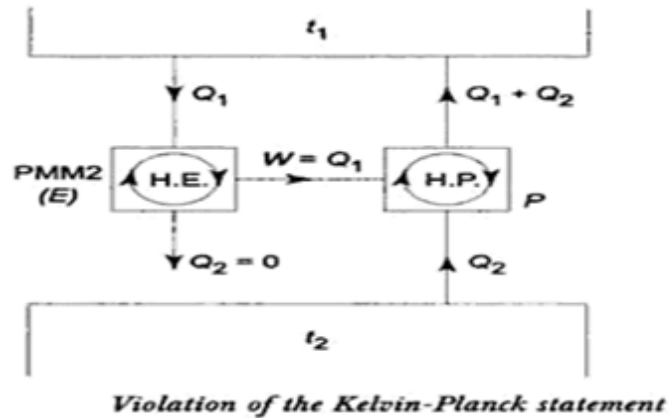
*Violation of the Clausius statement*

Let us assume a cyclic heat engine  $E$  operating between the same thermal energy reservoirs, producing  $W_{net}$  in one cycle. The rate of working of the heat engine is such that it draws an amount of heat  $Q_1$  from the hot reservoir equal to that discharged by the heat pump. Then the hot reservoir may be eliminated and the heat  $Q_1$  discharged by the heat pump is fed to the heat engine. So we see that the heat pump  $P$  and the heat engine  $E$  acting together constitute a heat engine

operating in cycles and producing net work while exchanging heat only with one body at a single fixed temperature. This violates the Kelvin-Planck statement.

(b) Let us now consider a perpetual motion machine of the second kind ( $E$ ) which produces net work in a cycle by exchanging heat with only one thermal energy reservoir (at  $t_1$ ) and thus violates the Kelvin-Planck statement

Let us assume a cyclic heat pump ( $P$ ) extracting heat  $Q_2$  from a low temperature reservoir at  $t_2$  and discharging heat to the high temperature reservoir at  $t_1$  with the expenditure of work  $W$  equal to what the PMM2 delivers in a complete cycle. So  $E$  and  $P$  together constitute a heat pump working in cycles and producing the sole effect of transferring heat from a lower to a higher temperature body, thus violating the Clausius statement.



From (a) and (b) it can be deduced that both statements are equivalent

Q1 b)

Entropy can be *transferred* to or from a system in two forms: *heat transfer* and *mass flow*.

(a) **Heat Transfer** Since  $dS = \frac{dQ_{\text{rev}}}{T}$ , when heat is added to a system  $dQ$  is positive, and the entropy of the system increases. When heat is removed from the system,  $dQ$  is negative, and the entropy of the system decreases.

Heat transferred to the system of fixed mass increases the internal energy of the system, as a result of which the molecules (of a gas) move with higher kinetic energy and collide more frequently, and so the disorder in the system increases. Heat is thus regarded as disorganised or disordered energy transfer which increases molecular chaos.

If heat  $Q$  flows reversibly from the system to the surroundings at  $T_0$ , the entropy increase of the surroundings is

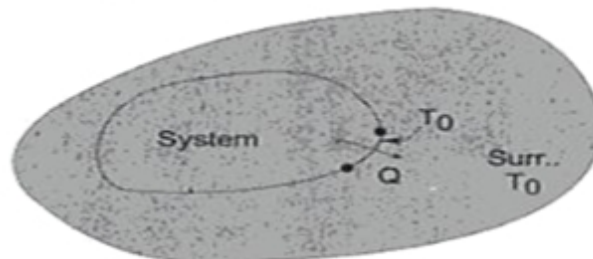
$$\Delta S_{\text{surr}} = \frac{Q}{T_0}$$

The entropy of the system is reduced by

$$\Delta S_{\text{sys}} = -\frac{Q}{T_0}$$

The temperature of the boundary where heat transfer occurs is the constant temperature  $T_0$ . It may be said that the system has lost entropy to the surroundings. Alternatively, one may state that the surroundings have gained

entropy from the system. Therefore, there is *entropy transfer* from the system to the surroundings along with heat flow. In other words, since the heat inflow increases the molecular disorder, there is flow of disorder along with heat. The sign of entropy transfer is the same as the sign of heat transfer: *positive*, if into the system, and *negative*, if out of the system.

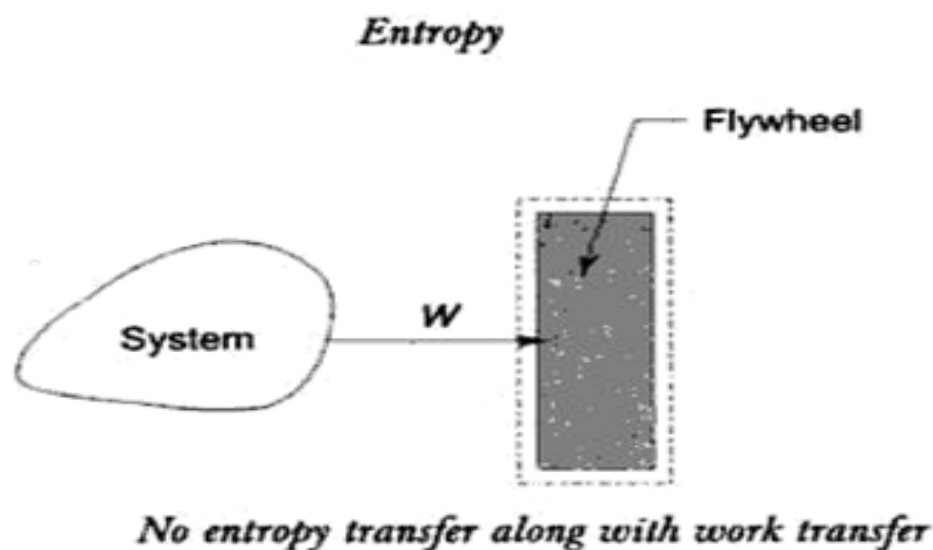


*Entropy transfer along with heat flow*

On the other hand, *there is no entropy transfer associated with work*. In the system delivers work to a flywheel, where energy is stored in a fully recoverable form. The flywheel molecules are simply put into rotation around the axis in a perfectly organised manner, and there is no dissipation and hence no entropy increase of the flywheel. The same can be said about work transfer in the compression of a spring or in the raising of a weight by a certain height. There is thus no entropy transfer along with work. If work is dissipated adiabatically into internal energy increase of the system there is an entropy increase in the system, but there is as such no entropy transfer to it.

Work is thus *entropy-free*, and no entropy is transferred with work. Energy is transferred with both heat and work, whereas entropy is transferred only with heat. The first law of thermodynamics makes no distinction between heat transfer and work. It considers them as *equals*. The distinction between heat transfer and work is brought about by the second law: *an energy interaction which is accompanied by entropy transfer is heat transfer, and an energy interaction which is not accompanied by entropy transfer is work. Thus, only energy is exchanged during work interaction, whereas both energy and entropy are exchanged during heat transfer.*

**(b) Mass Flow** Mass contains entropy as well as energy, and the entropy and energy of a system are proportional to the mass. When the mass of a system is doubled, so are the entropy and energy of the system. Both entropy and energy are carried into or out of a system by streams of matter, and the rates of entropy and energy transport into or out of a system are proportional to the mass flow rate. Closed systems do not involve any mass flow and thus any entropy transport. When an amount of mass  $m$  enters or leaves a system, an entropy of amount  $ms$ ,  $s$  being the specific entropy, accompanies it. Therefore, the entropy of a system increases by  $ms$  when the mass of amount  $m$  enters it, and decreases by the same amount when it leaves it at the same state.



Q2 a)

The efficiency of any heat engine cycle receiving heat  $Q_1$  and rejecting heat  $Q_2$  is given by

$$\eta = \frac{W_{\text{net}}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

By the second law, it is necessary to have a temperature difference ( $t_1 - t_2$ ) to obtain work of any cycle. We know that the efficiency of all heat engines operating between the same temperature levels is the same, and it is independent of the working substance. Therefore, for a reversible cycle (Carnot cycle), the efficiency will depend solely upon the temperatures  $t_1$  and  $t_2$ , at which heat is transferred, or

$$\eta_{\text{rev}} = f(t_1, t_2)$$

where  $f$  signifies some function of the temperatures.

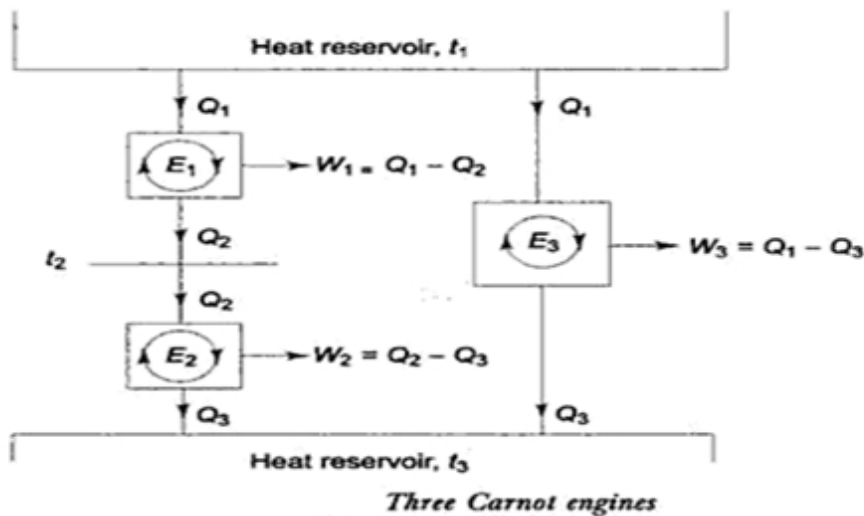
$$1 - \frac{Q_2}{Q_1} = f(t_1, t_2)$$

In terms of a new function  $F$

$$\frac{Q_1}{Q_2} = F(t_1, t_2)$$

If some functional relationship is assigned between  $t_1$ ,  $t_2$  and  $Q_1/Q_2$ , the equation becomes the definition of a temperature scale.

Let us consider two reversible heat engines,  $E_1$  receiving heat from the source at  $t_1$ , and rejecting heat at  $t_2$  to  $E_2$  which, in turn, rejects heat to the sink at  $t_3$



Now 
$$\frac{Q_1}{Q_2} = F(t_1, t_2); \quad \frac{Q_2}{Q_3} = F(t_2, t_3)$$

$E_1$  and  $E_2$  together constitute another heat engine  $E_3$  operating between  $t_1$  and  $t_3$ .

$\therefore \quad \frac{Q_1}{Q_3} = F(t_1, t_3)$

Now 
$$\frac{Q_1}{Q_2} = \frac{Q_1/Q_3}{Q_2/Q_3}$$

or 
$$\frac{Q_1}{Q_2} = F(t_1, t_2) = \frac{F(t_1, t_3)}{F(t_2, t_3)}$$

The temperatures  $t_1$ ,  $t_2$  and  $t_3$  are arbitrarily chosen. The ratio  $Q_1/Q_2$  depends only on  $t_1$  and  $t_2$ , and is independent of  $t_3$ . So  $t_3$  will drop out from the ratio on the right in equation

After it has been cancelled, the numerator can be written as  $\phi(t_1)$ , and the denominator as  $\phi(t_2)$ , where  $\phi$  is another unknown function. Thus



$$\frac{Q_1}{Q_2} = F(t_1, t_2) = \frac{\phi(t_1)}{\phi(t_2)}$$

Since  $\phi(t)$  is an arbitrary function, the simplest possible way to define the *absolute thermodynamic temperature*  $T$  is to let  $\phi(t) = T$ , as proposed by Kelvin. Then, by definition

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

The absolute thermodynamic temperature scale is also known as the *Kelvin scale*.

The heat absorbed  $Q_1$  and the heat rejected  $Q_2$  during the two reversible isothermal processes bounded by two reversible adiabatics in a Carnot engine can be measured. In defining the Kelvin temperature scale also, the triple point of water is taken as the standard reference point. For a Carnot engine operating between reservoirs at temperatures  $T$  and  $T_t$ ,  $T_t$  being the triple point of water arbitrarily assigned the value 273.16 K,

$$\frac{Q}{Q_t} = \frac{T}{T_t}$$

$$\therefore T = 273.16 \frac{Q}{Q_t}$$

If this equation is compared with the equations given in Article 2.3, it is seen that *in the Kelvin scale,  $Q$  plays the role of thermometric property*. The amount of heat supply  $Q$  changes with change in temperature, just like the thermal emf in a thermocouple.

It follows

$$T = 273.16 \frac{Q}{Q_t}$$

Q2 c)

The entropy of any closed system can increase in two ways:

- (a) by heat interaction in which there is entropy transfer
- (b) internal irreversibilities or dissipative effects in which work (or K.E.) is dissipated into internal energy increase.

If  $\delta Q$  is the infinitesimal amount of heat transferred to the system through its boundary at temperature  $T$ , the same as that of the surroundings, the entropy increase  $dS$  of the system can be expressed as

$$\begin{aligned}dS &= d_e S + d_i S \\&= \frac{\delta Q}{T} + d_i S\end{aligned}$$

where  $d_e S$  is the entropy increase due to external heat interaction and  $d_i S$  is the entropy increase due to internal irreversibility.

$$dS \geq \frac{\delta Q}{T}$$

$$\therefore d_i S \geq 0$$

The entropy increase due to internal irreversibility is also called entropy production or entropy generation,  $S_{\text{gen}}$ .

In other words, the entropy change of a system during a process is greater than the entropy transfer ( $\delta Q/T$ ) by an amount equal to the entropy generated during the process within the system ( $d_i S$ ), so that the *entropy balance* gives:

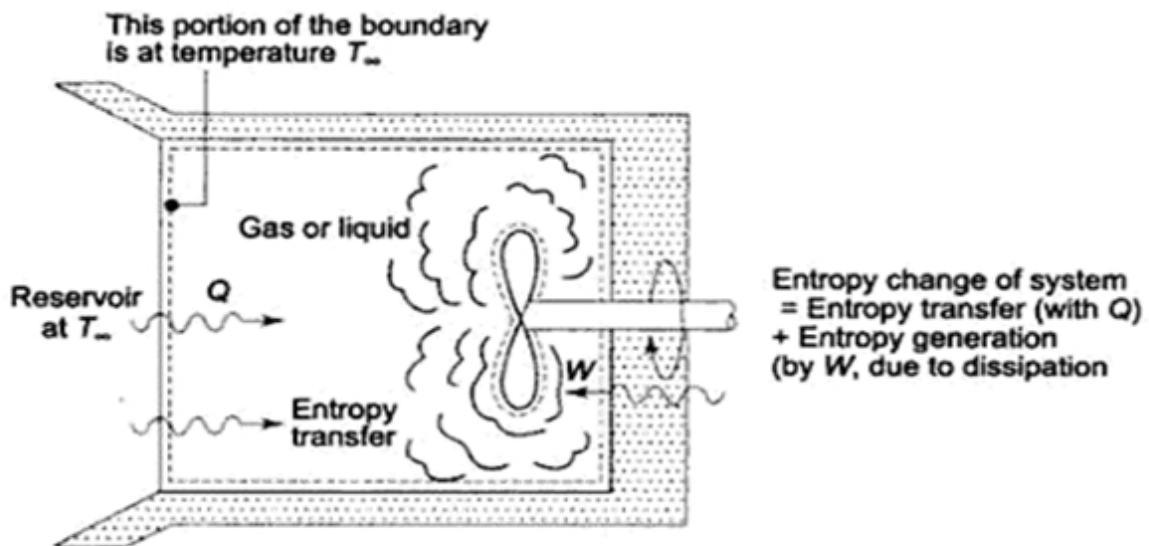
Entropy change = Entropy transfer + Entropy generation

$$\Delta S_{\text{system}} = \Delta S_{\text{transfer}} + \Delta S_{\text{gen}}$$

which is a verbal statement and illustrated

It may so happen that in a process (e.g., the expansion of a hot fluid in a turbine) the entropy decrease of the system due to heat loss to the surroundings

$\left(-\int \frac{\delta Q}{T}\right)$  is equal to the entropy increase of the system due to internal



*Illustration of the entropy transfer and entropy production concepts.*

irreversibilities such as friction, etc.  $(\int d_i S)$ , in which case the entropy of the system before and after the process will remain the same  $(\int dS = 0)$ . Therefore, an isentropic process need not be adiabatic or reversible.

But if the isentropic process is reversible, it must be adiabatic. Also, if the isentropic process is adiabatic, it cannot but be reversible. An adiabatic process need not be isentropic, since entropy can also increase due to friction etc. But if the process is adiabatic and reversible, it must be isentropic.

For an infinitesimal reversible process by a closed system,

$$\delta Q_R = dU_R + p dV$$

If the process is irreversible,

$$\delta Q_I = dU_I + \delta W$$

Since  $U$  is a property,

$$dU_R = dU_I$$

$$\therefore \delta Q_R - p dV = \delta Q_I - \delta W$$

$$\text{or} \quad \left( \frac{\delta Q}{T} \right)_R = \left( \frac{\delta Q}{T} \right)_I + \frac{p dV - \delta W}{T}$$

The difference  $(p dV - \delta W)$  indicates the work that is lost due to irreversibility, and is called the *lost work*  $\delta(LW)$ , which approaches zero as the process approaches reversibility as a limit. | can be expressed in the form

$$dS = d_e S + d_i S$$

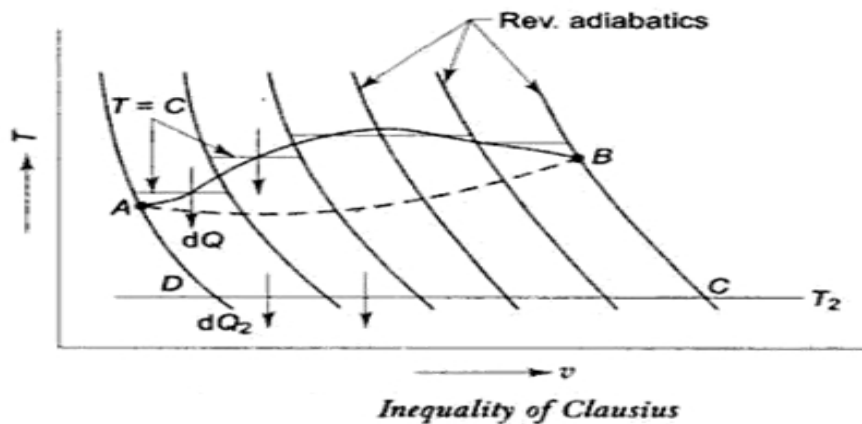
Thus the entropy of a closed system increases due to heat addition ( $d_e S$ ) and internal dissipation ( $d_i S$ ).



Q2 d)

Let us consider a cycle  $ABCD$

Let  $AB$  be a general process, either reversible or irreversible, while the other processes in the cycle are reversible. Let the cycle be divided into a number of elementary cycles, as shown. For one of these elementary cycles



$$\eta = 1 - \frac{dQ_2}{dQ}$$

where  $dQ$  is the heat supplied at  $T$ , and  $dQ_2$  the heat rejected at  $T_2$ .

Now, the efficiency of a general cycle will be equal to or less than the efficiency of a reversible cycle.

$$\therefore 1 - \frac{dQ_2}{dQ} \leq \left( 1 - \frac{dQ_2}{dQ} \right)_{\text{rev}}$$

$$\text{or } \frac{dQ_2}{dQ} \geq \left( \frac{dQ_2}{dQ} \right)_{\text{rev}}$$

$$\text{or } \frac{dQ}{dQ_2} \leq \left( \frac{dQ}{dQ_2} \right)_{\text{rev}}$$

$$\text{Since } \left( \frac{dQ}{dQ_2} \right)_{\text{rev}} = \frac{T}{T_2}$$

$$\therefore \frac{dQ}{dQ_2} \leq \frac{T}{T_2}$$

$$\text{or } \frac{dQ}{T} \leq \frac{dQ_2}{T_2}, \text{ for any process } AB, \text{ reversible or irreversible.}$$

For a reversible process

$$ds = \frac{dQ_{\text{rev}}}{T} = \frac{dQ_2}{T_2}$$

Hence, for any process  $AB$

$$\frac{dQ}{T} \leq ds$$

For any infinitesimal process undergone by a system, we have  
for the total mass

$$dS \geq \frac{dQ}{T}$$

For an isolated system which does not undergo any energy interaction with the surroundings,  $dQ = 0$ .

Therefore, for an isolated system

$$dS_{\text{iso}} \geq 0$$

For a reversible process,

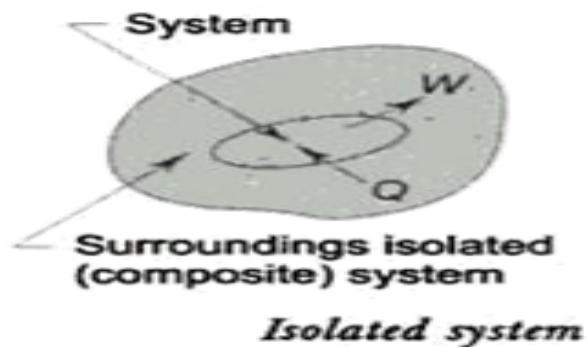
$$dS_{\text{iso}} = 0$$

or  $S = \text{constant}$

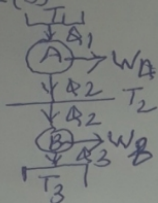
For an irreversible process

$$dS_{\text{iso}} > 0$$

It is thus proved that *the entropy of an isolated system can never decrease*. It always increases and remains constant only when the process is reversible. This is known as the *principle of increase of entropy*, or simply the *entropy principle*. It is the quantitative general statement of second law from the macroscopic viewpoint.



Q2(b)



Given:  $T_1 = 421 + 273 = 694\text{K}$

$T_3 = 4.4 + 273 = 277.4\text{K}$

$Q_1 = 200\text{KJ}$   $W_A = 2W_B$

To find: (i)  $T_2$  (ii)  $\eta_A, \eta_B$  (iii)  $Q_3$

Sol: Efficiency of heat engine is given by-

$$\eta_A = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1} = \frac{W_A}{Q_1}$$

$$\eta_B = \frac{Q_2 - Q_3}{Q_2} = \frac{T_2 - T_3}{T_2} = \frac{W_B}{Q_2}$$

(i) Since  $W_A = 2W_B$   $\therefore T_1 - T_2 = 2(T_2 - T_3)$

$$694 - T_2 = 2(T_2 - 277.4)$$

$$T_2 = 416.27\text{K or } 143.27^\circ\text{C}$$

(ii)  $\eta_A = \frac{694 - 416.27}{694} \times 100 = 40.01\%$

$\eta_B = \frac{416.27 - 277.4}{416.27} \times 100 = 33.36\%$

(iii)

$$\eta_A = \frac{Q_1 - Q_2}{Q_1} \quad \text{i.e. } 0.4001 = \frac{200 - Q_2}{200}$$

$$Q_2 = 119.98\text{KJ}$$

$$\eta_B = \frac{Q_2 - Q_3}{Q_2} \quad \text{i.e. } 0.3336 = \frac{119.98 - Q_3}{119.98}$$

$$Q_3 = 79.95\text{KJ}$$