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Mid Semester Examination – October – 2017
Model Answer of Analysis and Synthesis of Mechanism

Branch: M.Tech (Design Engineering) **Sem.:- I**

Subject with Subject Code: - MMD102 **Marks:20**

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Q.No.1 Explain briefly any one of the following

a.) Euler Savary Equation (08 Marks)

Euler savary equation gives the radius of curvature and the centre of curvature of a coupler curve in rather direct fashion. The so-called inflection circle shows the location of coupler points whose curves have an infinite radius of curvature.

When two rigid bodies move relative to each other with planar motion, any arbitrarily chosen point *A* of one describes a path or locus relative to a coordinate system fixed to the other. At any given instant there is a point *A′*, attached to the other body, which is the center of curvature of the locus of *A*. If we take the kinematic inversion of this motion, *A′* also describes a locus relative to the body containing *A*, and it so happens that *A* is the center of curvature of this locus. Each point therefore acts as the center of curvature of the path traced by the other, and the two points are called *conjugates* of each other. The distance between these two conjugate points is the radius of curvature of either locus. Figure 4.24 shows two circles with centers at C and *C′*. Let us think of the circle with center C' as the fixed centrode and think of the circle with center C as the moving centrode of two bodies experiencing some particular relative planar motion. In actuality, the fixed centrode need not be fixed but is attached to the body that contains the path whose curvature is sought. Also, it is not necessary that the two centrodes be circles; we are interested only in instantaneous values and, for convenience, we will think of the centrodes as circles matching the curvatures of the two actual centrodes in the region near their point of contact *P*. W

hen the bodies containing the two centrodes move relative to each other, the centrodes appear to roll against each other without slip. Their point of contact *P*, of course, is the instant center of velocity. Because of these properties, we can think of the two circular centrodes as actually representing the shapes of the two moving bodies if this helps in visualizing the motion.

If the moving centrode has some angular velocity ω relative to the fixed centrode, the instantaneous velocity* of point C is

$$V_C = \omega R_{CP} \qquad (a)$$

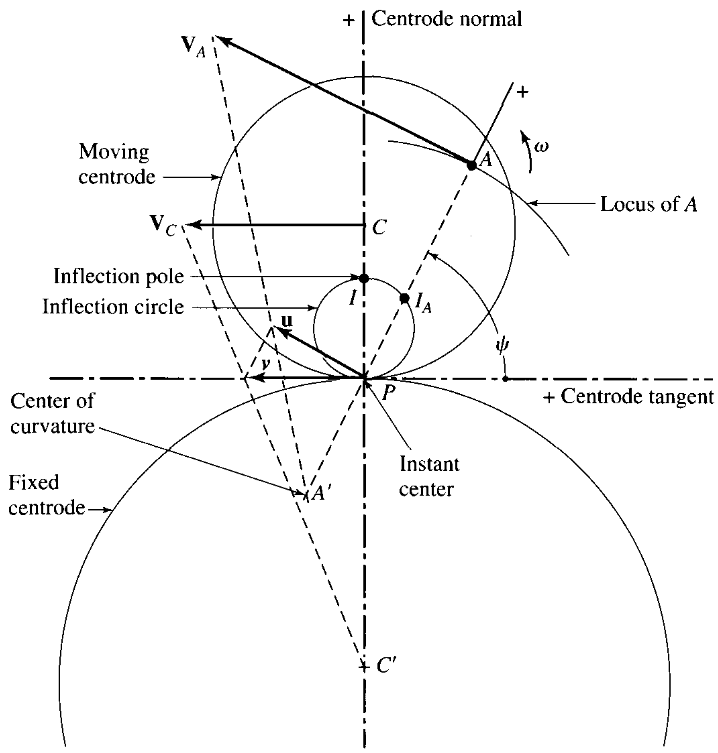


Figure 4.24 The Hartmann construction.

Similarly, the arbitrary point A, whose conjugate point A' we wish to find, has a velocity of

$$V_A = \omega R_{AP} \tag{b}$$

As the motion progresses, the point of contact of the two centrodes, and therefore the location of the instant center P, moves along both centrodes with some velocity $\boldsymbol{v}$. As shown in Fig. 4.24, $\boldsymbol{v}$ can be found by connecting a straight line from the terminus of V_C to the point C'. Alternatively, its size can be found from

$$v = \frac{R_{PC'}}{R_{CC'}} V_C \tag{c}$$

A graphical construction for A', the center of curvature of the locus of point A, is shown in Fig. 4.24 and is called the *Hartmann constuction*. First the component $\boldsymbol{u}$ of the instant center's velocity $\boldsymbol{v}$ is found as that component parallel to $\mathbf{V}_A$ or perpendicular to $\mathbf{R}_{AP}$. Then the intersection of the line AP and a line connecting the termini of the velocities $\mathbf{V}_A$ and $\boldsymbol{u}$ gives the location of the conjugate point A'. The radius of curvature ρ of the locus of point A is $\rho = R_{AA'}$.

An analytical expression for locating point A' would also be desirable and can be derived from the Hartmann construction. The magnitude of the velocity $\boldsymbol{u}$ is given by

$$u = v \sin \psi \tag{d}$$

where ψ is the positive counterclockwise angle measured from the centrode tangent to the line of $\mathbf{R}_{AP}$. Then, noticing the similar triangles in Fig. 4.24, we can write

$$u = \frac{R_{PA'}}{R_{AA'}} V_A \tag{e}$$

Now, equating the expressions of Eqs. (d) and (e) and substituting from Eqs. (a), (b), and (c) gives

$$u = \frac{R_{PC'}R_{CP}}{R_{CC'}}\omega \sin\psi = \frac{R_{PA'}R_{AP}}{R_{AA'}}\omega \tag{f}$$

Dividing by $\omega \sin\psi$ and inverting, we obtain

$$\frac{R_{AA'}}{R_{AP}R_{PA'}}\sin\psi = \frac{R_{CC'}}{R_{CP}R_{PC'}} = \frac{\omega}{v} \tag{g}$$

Next, upon noticing that $\mathbf{R}_{AA'} = \mathbf{R}_{AP} - \mathbf{R}_{A'P}$ and $\mathbf{R}_{CC'} = \mathbf{R}_{CP} - \mathbf{R}_{C'P}$, we can reduce this equation to the form

$$\left(\frac{1}{R_{AP}} - \frac{1}{R_{A'P}}\right)\sin\psi = \left(\frac{1}{R_{CP}} - \frac{1}{R_{C'P}}\right) \tag{4.45}$$

This important equation is one form of the *Euler–Savary equation*. Once the radii of curvature of the two centrodes R_{CP} and $R_{C'P}$ are known, this equation can be used to determine the positions of the two conjugate points A and A' relative to the instant center P.

Before continuing, an explanation of the sign convention is important. In using the Euler–Savary equation, we may arbitrarily choose a positive sense for the centrode tangent; the positive centrode normal is then 90° counterclockwise from it. This establishes a positive direction for the line CC' which may be used in assigning appropriate signs to R_{CP} and $R_{C'P}$. Similarly, an arbitrary positive direction can be chosen for the line AA'. The angle ψ is then taken as positive counterclockwise from the positive centrode tangent to the positive sense of line AA'. The sense of line AA' also gives the appropriate signs for R_{AP} and $R_{A'P}$ for Eq. (4.45).

There is a major drawback to the above form of the Euler–Savary equation in that the radii of curvature of both centrodes, R_{CP} and $R_{C'P}$, must be found. Usually they are not known, any more than the curvature of the locus itself was known. However, this difficulty can be overcome by seeking another form of the equation.

Let us consider the particular point labeled I in Fig. 4.24. This point is located on the centrode normal at a location defined by

$$\frac{1}{R_{IP}} = \frac{1}{R_{CP}} - \frac{1}{R_{C'P}} \tag{h}$$

If this particular point is chosen for A in Eq. (4.45), we find that its conjugate point I' must be located at infinity. The radius of curvature of the path of point I is infinite, and the locus of I therefore has an inflection point at I. The point I is called the *inflection pole.*

Let us now consider whether there are any other points A of the moving body which also have infinite radii of curvature at the instant considered. If so, then for each of these points $R_{I_AP} = \infty$ and, from Eqs. (4.45) and (h),

$$R_{I_AP} = R_{IP}\sin\psi \tag{4.46}$$

This equation defines a circle called the *inflection circle,* whose diameter is R_{IP}, as shown in Fig. 4.24. Every point on this circle has its conjugate point at infinity, and each therefore has an infinite radius of curvature at the instant shown.

Now, with the help of Eq. (4.46), the Euler–Savary equation can be written in the form

$$\frac{1}{R_{AP}} - \frac{1}{R_{A'P}} = \frac{1}{R_{I_A P}} \tag{4.47}$$

Also, after some further manipulation, this can be put in the form

$$\rho = R_{AA'} = \frac{R_{AP}^2}{R_{AI_A}} \tag{4.48}$$

b.) Bobillier construction (08 Marks)

The Hartmann construction provides one graphical method of finding the conjugate point and the radius of curvature of the path of a moving point, but it requires knowledge of the curvature of the fixed and moving centrodes. It would be desirable to have graphical methods of obtaining the inflection circle and the conjugate of a given point without requiring the curvature of the centrodes. Such graphical solutions are presented in this section and are called the Bobillier constructions.To understand these constructions, consider the inflection circle and the centrode normal N and centrode tangent T shown in Fig. below. Let us select any two points A and B of the moving body which are not on a straight line through P. Now, by using the Euler-Savary equation, we can find the two corresponding conjugate points A' and B'. The intersection of the lines AB and A' B' is labeled Q. Then, the straight line drawn through P and Q is called the collineation axis. This axis applies only to the two lines AA' and BB' and so is said to belong to these two rays; also, the point Q will be located differently on the collineation axis if another set of points A and B is chosen on the same rays. Nevertheless, there is a unique relationship between the collineation axis and the two rays used to define it. This relationship is expressed in Bobillier's theorem, which states that the angle from the centrode tangent to one of these rays is the negative of the angle from the collineation axis to the other ray.

In applying the Euler-Savary equation to a planar mechanism, we can usually find two pairs of conjugate points by inspection, and from these we wish to determine the inflection circle graphically. For example, a four-bar linkage with a crank 02A and a follower 04B has A and O2 as one set of conjugate points and Band 04 as the other, when we are interested in the motion of the coupler relative to the frame. Given these two pairs of conjugate points, how do we use the Bobillier theorem to find the inflection circle?

let A and A' and Band B' represent the known pairs of conjugate points. Rays constructed through each pair intersect at P, the instant center of velocity, giving one point on the inflection circle. Point Q is located next by the intersection of a ray through A and B with a ray through A' and B'. Then the collineation axis can be drawn as the line PQ. Drawing a straight line through P parallel to A' B', we identify the point W as the intersection of this line with the line AB. Now, through W we draw a second line parallel to the collineation axis.

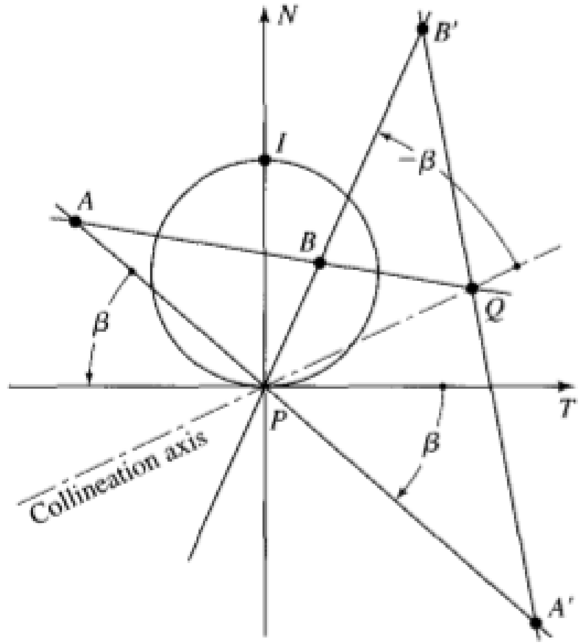


Fig. 4.27 Bobillier construction

This line intersects AA' at IA and BB' at Is, the two additional points on the inflection circle for which we are searching. We could now construct the circle through the three points IA, Is, and P, but there is an easier way. Remembering that a triangle inscribed in a semicircle is a right triangle having the diameter as its hypotenuse, we erect a perpendicular to A P at IA and another perpendicular to B P at Is. The intersection of these two perpendiculars gives point I, the inflection pole. Because PI is the diameter, the inflection circle, the centrode normal N, and the centrode tangent T can all be easily constructed. To show that this construction satisfies the Bobillier theorem, note that the arc from P to IA is inscribed by the angle that lAP makes with the centrode tangent. But this same arc is also inscribed by the angle PIsIA. Therefore these two angles are equal.

Q.No. 2 Write short note on (Attempt any three of the following)

a.) Method of Normal Accelerations (04 Mark)

This method is applicable only to mechanisms having a low degree of complexity. It is also useful as a supplement to the auxiliary-point method for certain mechanisms with a high degree of complexity when the latter method alone is not sufficient. The underlying principle of the method is that the acceleration component of a point *P* on a constrained link, in a direction perpendicular to its velocity (called normal component), is independent of the angular acceleration of the link.

The steps to be followed in applying this method are:

1.Transform the mechanism into a simple one by changing the input link.

2. Carry out the velocity analysis with this alternative input link, and determine the true velocities.

3. Draw an auxiliary *acceleration diagram* based on true velocities and zero acceleration of the alternative input link. Determine the normal component of acceleration of the floating point which has a path of unknown radius of curvature.

4. Construct the true *acceleration* diagram with the actual input acceleration, using the information obtained in steps 1-3.

b.) Auxiliary Point Method (04 Mark)

The auxiliary-point method is very powerful and is applicable to all low-complexity mechanism: and to most high-complexity mechanisms. In certain cases of high-complexity mechanisms, this method alone may not be sufficient and has to be used in conjunction with the method of normal components.

In this method, the auxiliary points are determined on the higher-order floating link at the intersection of auxiliary lines drawn through the motion-transfer point of the link in directions along which components of velocities and accelerations can be obtained. Two such auxiliary points are sufficient. The velocity and acceleration components of these two auxiliary points are obtained in two auxiliary directions (along which they lie). Thereby, the vector itself can he obtained. Once the velocities and accelerations of the two auxiliary points have been determined, the analysis can be completed with the help of images of the floating links.

c.) Fixed and Moving Centrodes (04 Mark)

The location of an instant center of velocity is defined only instantaneously and changes as the mechanism moves. When the changing locations of an instant center are found for all possible phases of the mechanism, they describe a curve or locus, called a *centrode.** In Fig. 3.38, the instant center P_{13} is located at the intersection of the extension of links 2 and 4. As the linkage is moved through all possible positions, P_{13} traces out the curve called the *fixed centrode* on link 1.

Figure 3.39 shows the inversion of the same linkage in which link 3 is fixed and link 1 is movable. When this inversion is moved through all possible positions, P_{13} traces a Different curve on link 3. For the original linkage, with link 1fixed, this is the curve traced by PI3 on the coordinate system of the moving link 3; it is called the moving centrode.

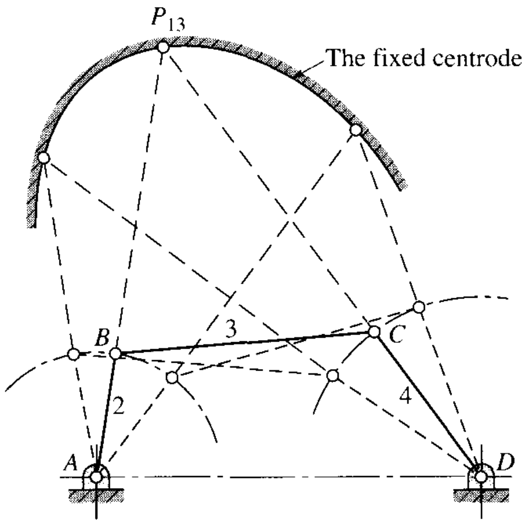


Figure 3.38 The fixed centrode.

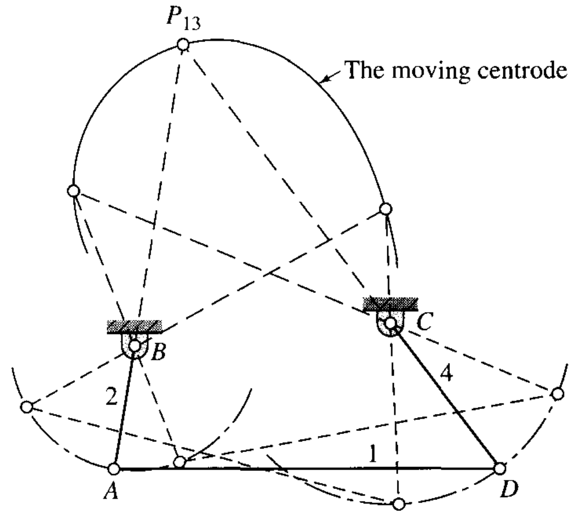


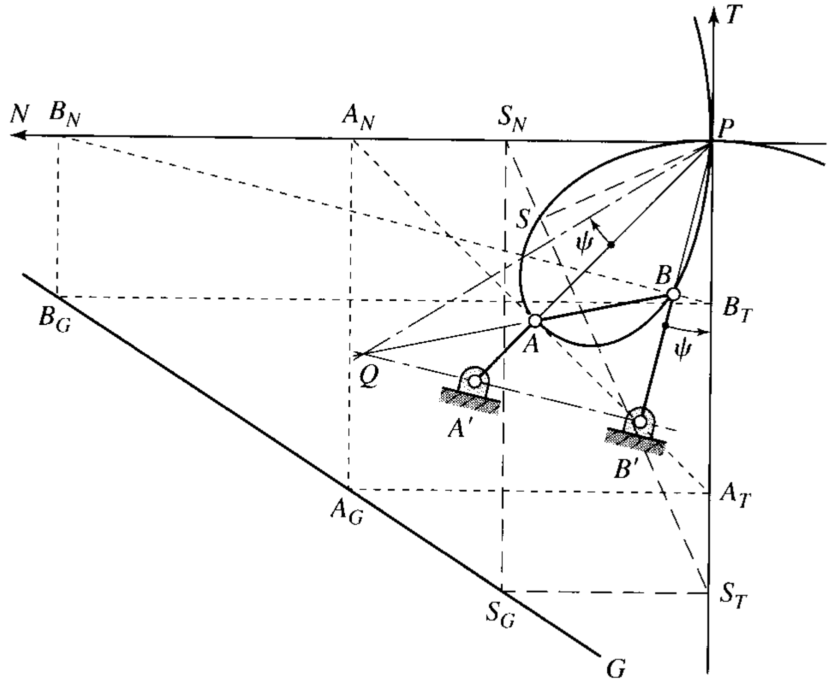
Figure 3.39 The moving centrode.

d.) Cubic of Stationary Curvature (04 Mark)

Consider a point on the coupler of a planar four-bar linkage that generates a path relative to the frame whose radius of curvature, at the instant considered, is p. For most cases, because the coupler curve is of sixth order, this radius of curvature changes continuously as the point moves. In certain situations, however, the path will have stationary curvature, which means that

$$\frac{d\rho}{ds} = 0$$

where s is the increment traveled along the path. The locus of all points on the coupler or moving plane which have stationary curvature at the instant considered is called the cubic of stationary curvature or sometimes the circling-point curve. It should be noted that stationary curvature does not necessarily mean constant curvature, but rather that the continually varying radius of curvature is passing through a maximum or minimum. Here we will present a fast and simple graphical method for obtaining the cubic of stationary curvature. In Fig. below we have a four-bar linkage A' AB B', with A' and B' the frame pivots. Then points A and B have stationary curvature-in fact, constant curvature about centers at A' and B'; hence, A and B lie on the cubic. The first step of the construction is to obtain the centrode normal and centrode tangent. Because the inflection circle is not needed, we locate the collineation axis PQ as shown and draw the centrode tangent T at the angle $_{1/1}$ from the line PA' to the collineation axis. This construction follows directly from Bobillier's theorem. We also construct the centrode normal N. At this point it may be convenient to reorient the drawing on the working surface so that the T-square or horizontal lies along the centrode normal. Next we construct a line through A perpendicular to PA and another line through B perpendicular to PB. These lines intersect the centrode normal and centrode tangent at AN, AT and BN, BT, respectively, as shown in fig. Now we draw the two rectangles PAN AG A T and P BNBG BT; the points AG and BG define an auxiliary line G that we will use to obtain other points on the cubic. Next we choose any point SG on the line G. A ray parallel to N locates ST, and another ray parallel to T locates SN.



Connecting ST with SN and drawing a perpendicular to this line through P locates point S, another point on the cubic of stationary curvature. We now repeat this process as often as desired by choosing different points on G, and we draw the cubic as a smooth curve through all the points S obtained.