DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE – RAIGAD -402 103

Mid Semester Examination – October - 2017

Branch: All courses **Sem.:- I**

Subject with Subject Code:- Engineering Mathematics –I (MATH101) **Marks: 20**

Date:-03/10/2017 **Time:- 1 Hr.**

MODEL SOLUTION

Q.N.	Sub.-Q.N.		Marks
1.	a)	iii) 4	01
	b)	i) 2	01
	c)	i) The elements of its principle diagonal	01
	d)	iv) $X_1^T X_2 = I$	01
	e)	iii) 0	01
	f)	ii) $x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$	01
2.	a)	Step- :To find the eigen values - We know the charters tics equation of matrix A in is $\vert A - \lambda I\vert = 0$ $$\lambda^3 - S_1\lambda^2 + S_2\lambda - \vert A\vert = 0$$ $$\lambda^3 - 8\lambda^2 + 4\lambda + 48 = 0$$ solving we get $\lambda = -2, 4, 6$	01
		Step-II: To find corresponding eigen vectors- Case-I: For $\lambda = -2$ the matrix equation $[A - \lambda I]X = 0$ reduces $$\begin{bmatrix} 7 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ By $R_1 and\ R_3$ we have $$7x + 0y + z = 0$$ $$x + 0y + 7z = 0$$	01
		By Crammers rule $\frac{\mathrm{x}}{0} = \frac{-\mathrm{y}}{48} = \frac{\mathrm{z}}{0}$ this gives $X_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ Case-II: For $\lambda = 4$ the matrix equation $[A - \lambda I]X = 0$ reduces	01
		$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -6 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ By $R_1 and\ R_2$ we have $x + 0y + z = 0$ and $x - 6y + 0z = 0$ By Crammers rule $\frac{\mathrm{x}}{6} = \frac{-\mathrm{y}}{0} = \frac{\mathrm{z}}{-6}$ this gives$X_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$	01
		Case-III: For $\lambda = 6$ the matrix equation $[A - \lambda I]X = 0$ reduces	

		$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -8 & 0 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ By $R_1 and\ R_2$ we have $-1x + 0y + z = 0$ $0x - 8y + 0z = 0$ By Crammers rule $\frac{\mathrm{x}}{8} = \frac{-\mathrm{y}}{0} = \frac{\mathrm{z}}{8}$ this gives $X_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$	01 01
	b)	Given $y = (\sin^{-1} x)^2$ ---(i) Differentiating w.r.t x and squaring we get $(1 - x^2)y_1 = 4(\sin^{-1} x)^2 = 4y$ ------------------------ ---------(ii) again differentiating w.r.t x and squaring we get $(1 - x^2)y_2 - xy_1 = 2$ ---(iii) Apply Leibnitz's rule for nth differentiation $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$ ------------------(iv) Put $x = 0$ in all above four equations, this gives $y(0) = 0$, $y_1(0) = 0,$ $y_2(0) = 2$ and $y_{n+2}(0) = n^2y_n(0)$ -(v) Put $n = 1,2,3 \dots$ in equation (v) this gives $y_3(0) = 0,$ $y_4(0) = 2^2 2,$ $y_5(0) = 0,$ $y_6(0) = 4^2 2^2 2$ an so on Put all these in Maclaurin's Theorem $y(x) = y(0) + xy_1(0) + \frac{x^2}{2!}y_2(0) + \frac{x^3}{3!}y_3(0) + \cdots \dots$ $(\sin^{-1} x)^2 = 2\frac{x^2}{2!} + 2\ 2^2\frac{x^4}{4!} + 2\ 2^2 4^2\frac{x^6}{6!} + \cdots \dots.$	01 01 01 01 01 01
3.	a)	The given system can be written as $\begin{bmatrix} 2 & -3 & 6 & -5 \\ 0 & 1 & -4 & 1 \\ 4 & -5 & 8 & -9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ k \end{bmatrix}$ $[A{:}B] = \begin{bmatrix} 2 & -3 & 6 & -5 & 3 \\ 0 & 1 & -4 & 1 & 1 \\ 4 & -5 & 8 & -9 & k \end{bmatrix}$ By applying the row transformations $R_3 - 2R_1$, $R_3 - R_2$ we get $= \begin{bmatrix} 2 & -3 & 6 & -5 & 3 \\ 0 & 1 & -4 & 1 & 1 \\ 0 & 0 & 0 & 0 & k-7 \end{bmatrix}$ It is clear that $\rho(A) = 2$ (i)For no solution $\rho(A) \neq \rho(A{:}B)$. This will require $k - 7 \neq 0$ or $k \neq 7$ (ii)For infinite no. of solution $\rho(A) = \rho(A{:}B) < n(= 4)$. This will require $k - 7 = 0$ or $k = 7$	01 01 01 01
	b)	From given equation $y = b\cos[n(n\ln x - n\ln n)]$-------------------------------(i)	01

		Differentiating w.r.t x $xy_1 = -bn\sin[n(n\ln x - n\ln n)]$ ---------------------- (ii) Differentiating again w.r.t x $x^2y_2 + y_1 = -bn^2\cos[n(n\ln x - n\ln n)] = -n^2y$ ------------(iii) Apply Leibnitz's rule for nth differential and collect the similar terms $x^2y_{n+2} + (2n+1)xy_{n+1} + 2n^2y_n = 0$ ------------------(iv)	01 01 01
	c)	We know Taylor theorem state that $f(x+h) = f(x) + h\,f'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \cdots$ (A) $Here\ f(x+h) = \tan^{-1}(x+h)$ and $x = 1$ &$h = 0.003$ $f(x) = \tan^{-1}(x)$ Differentiating successively w.r.t x we get $f'(x) = \frac{1}{1+x^2}$ $f''(x) = -\frac{2x}{(1+x^2)^2}$ $f'''(x) = -\frac{2(1-3x^2)}{(1+x^2)^3}$ Put $x = 1$ in all above this gives $f(1) = \frac{\pi}{4} f'(1) = \frac{1}{2}$ $f''(1) = -\frac{1}{2} f'''(1) = \frac{1}{2}$ Putting all these values in (A) we get $\tan^{-1}(1.003) = \frac{\pi}{4} + (0.003)\frac{1}{2} + \frac{(0.003)^2}{2!}\left(\frac{-1}{2}\right) + \frac{(0.003)^3}{3!}\left(\frac{1}{2}\right) + \cdots$ = 0.78690	01 01 01 01