

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

SEMESTER EXAMINATION: ~~MAY~~ - 2017

Mechanical/Electrical/ExTC/Chemical/Petrochemical/Computer/IT/Civil

Subject : Engineering Mathematics-I (New)

Time : 03 Hrs

BSH101

Semester : I

Max. Marks : 60

2 MAY 2017

INSTRUCTION: ATTEMPT ANY FIVE QUESTIONS.

Q1. (a) Find the rank of the matrix  $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 5 & 4 \end{bmatrix}$  by reducing it to normal form. [4 Marks]

(b) For what values of  $k$  is the following system of equations consistent, and hence solve for them: [4 Marks]

$$x + y + z = 1; x + 2y + 4z = k; x + 4y + 10z = k^2.$$

(c) Find the eigen values and eigen vectors of the matrix  $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ . [4 Marks]

Q2. (a) Find the  $n^{th}$  derivative of  $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$  in terms of  $r$  and  $\theta$ . [4 Marks]

(b) If  $y = (x^2 - 1)^n$ , prove that  $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$ . [4 Marks]

(c) Expand  $f(x+h) = \tan^{-1}(x+h)$  in powers of  $h$  and hence find the value of  $\tan^{-1}(1.003)$  upto five places of decimal. [4 Marks]

Q3. (a) If  $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$ , prove that  $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right]$ . [4 Marks]

(b) If  $z$  is a homogeneous function of degree  $n$  in  $x, y$ , then prove that [4 Marks]

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$

(c) If  $F = F(x, y, z)$  where  $x = u + v + w$ ,  $y = uv + vw + wu$ ,  $z = xyz$ , then show that [4 Marks]

$$u \frac{\partial F}{\partial u} + v \frac{\partial F}{\partial v} + w \frac{\partial F}{\partial w} = x \frac{\partial F}{\partial x} + 2y \frac{\partial F}{\partial y} + 3z \frac{\partial F}{\partial z}.$$

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Q4. (a) Expand  $f(x, y) = \cos x \sin y$  as far as the terms of third degree. [4 Marks]

(b) If the sides and angles of a plane triangle vary in such a way that its circum-radius remains constant, prove that  $\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos C} = 0$ , where  $da, db, dc$  are smaller increments in the sides  $a, b, c$  respectively. [4 Marks]

(c) Find the maximum and minimum distances from the origin to the curve  $3x^2 + 4xy + 6y^2 = 140$ . [4 Marks]

Q5. (a) Change to polar co-ordinates and evaluate  $I = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ . [4 Marks]

(b) Evaluate  $I = \int_0^1 \int_{\sqrt{x}}^1 \frac{y^2}{\sqrt{y^4-x^2}} dx dy$  by changing the order of integration. [4 Marks]

(c) Evaluate  $I = \int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z-x^2}} dy dx dz$ . [4 Marks]

Q6. (a) Find the interval of convergence of the series  $\sum \frac{n!}{n^n} x^n$ . [4 Marks]

(b) Test the convergence of the series  $\sum \frac{(n+1)^n}{n^{n+1}} x^n$ . [4 Marks]

(c) Test the convergence of the series  $\sum_{n=1}^\infty \frac{n!}{(n^n)^2}$ . [4 Marks]

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